

Vehicle Routing Problems with Time Window

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Travelling Salesman Problem

- A travelling salesman has to visit a number of cities. He knows the cost of travel between each pair.

What order does he visit the cities to minimise cost?

- Long and well-studied problem (since the 60's)
- A sub-problem in many others
- Used in chip fabrication and many other real-world problems

Vehicle Routing Problem

More than 50 years ago, Dantzig and Ramser introduce the Vehicles Routing Problem in 1959

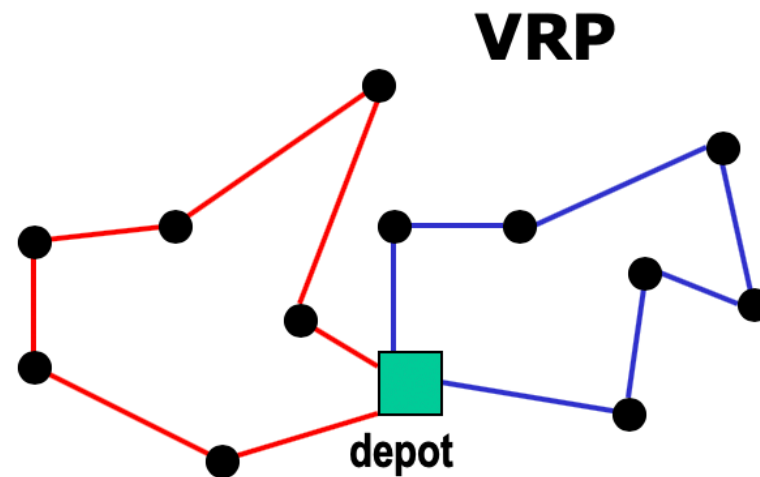
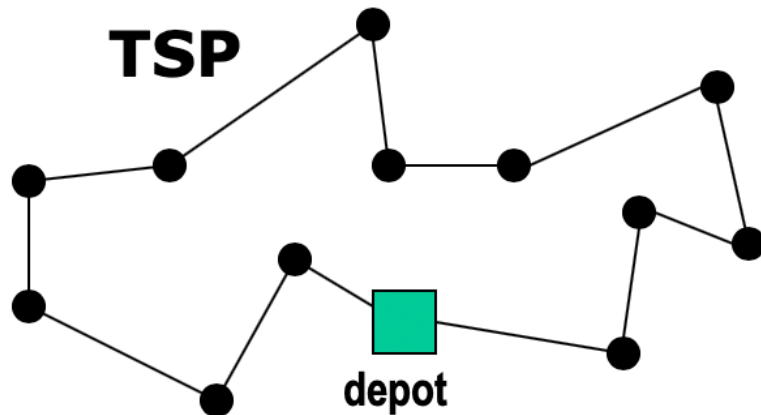
The VRP is the generic name given to all the set of problems in which set of routes for a fleet of vehicles at one or several depots

The objective of the VRP is to form a route with the lowest cost to serve all the customers

There are many real world applications for VRP, especially in logistic system

TSP & VRP

- TSP: One route can serve all orders
- VRP: More than one route is required to serve all orders



Pure Pickup or Delivery

- Pickup: Design route to pick up orders from many customers and deliver to depot
- Delivery: Load vehicles at depot. Design route to deliver to many customers
- Mixed Pickup & Delivery
- Examples:
 - UPS, FedEx, etc.
 - Manufacturers & carriers.
 - Carpools, school buses, etc

Time Window Constraints

- A window during which service can start
- E.g. a customer can only accept delivery between 3:00pm to 5:00pm
- Bank deliveries, postal deliveries and school bus routing
- **Hard time window**: the time window constraint must be satisfied
- **Soft time window**: the time window constraint can be violated.
- The violation of soft constraints is usually penalized and added to the objective function.

Other Constraints

- Capacity constraints for vehicles, vehicles may have different capacities
- Each customer can only be visited once
- The vehicles must start and end at the depot

Variables

- $N = \{1, \dots, n\}$: set of requests. 0 and $n + 1$ represent the starting and ending node for all paths, which in our case is a single location
 - K : number of available vehicles to be routed and scheduled
 - q_i : for each request $i \in N$, there is a known quantity q_i
 - $[e_i, l_i]$: for each request $i \in N$, there is a time window $[e_i, l_i]$
 - s_i : for each request $i \in N$, there is a service time s_i
- $[E^k, L^k]$: every vehicle k has a working time interval $[E^k, L^k]$, $[e_{0(k)}, l_{0(k)}] = [e_{n+1(k)}, l_{n+1(k)}] = [E^k, L^k]$
 - $G = (V, A)$: consider the graph $G = (V, A)$, where the set of nodes is equal to $V = N \cup \{0, n + 1\}$, and the set A contains all feasible arcs, that is $A \subseteq V \times V$
 - c_{ij} : for each arc $(i, j) \in A$, there is a cost c_{ij}
 - t_{ij} : for each arc $(i, j) \in A$, there is a travel time t_{ij}
 - Q^k : the capacity of vehicle k
 - X_{ij}^k :
$$X_{ij}^k = \begin{cases} 1 & \text{arc } (i, j) \text{ is used by vehicle } k \\ 0 & \text{otherwise} \end{cases}$$
 - T_i^k : the start time of service at node i by vehicle k
 - L_i^k : the load of the vehicle k after servicing node i

Mathematical Formulation

$$\min \sum_{k \in K} \sum_{(i,j) \in A} c_{ij} X_{ij}^k \quad (1)$$

$$\text{s.t.} \quad \sum_{k \in K} \sum_{j \in N \cup \{n+1\}} X_{ij}^k = 1 \quad \forall i \in N \quad (2)$$

$$\sum_{k \in K} \sum_{j \in N} X_{0,j}^k \leq K \quad (3)$$

$$\sum_{j \in N \cup \{n+1\}} X_{0j}^k = 1 \quad \forall k \in K \quad (4)$$

$$\sum_{i \in N \cup \{0\}} X_{ij}^k - \sum_{i \in N \cup \{n+1\}} X_{ji}^k = 0 \quad \forall k \in K, \forall j \in N \quad (5)$$

$$\sum_{i \in N \cup \{0\}} X_{i,n+1}^k = 1 \quad \forall k \in K \quad (6)$$

$$X_{ij}^k (T_i^k + s_i + t_{ij}^k - T_j^k) \leq 0 \quad \forall k \in K, (i,j) \in A \quad (7)$$

$$e_i \leq T_i^k \leq l_i \quad \forall k \in K, \forall i \in V \quad (8)$$

$$X_{ij}^k (L_i^k + q_i - L_j^k) = 0 \quad \forall k \in K, (i,j) \in A \quad (9)$$

$$L_i^k \leq Q^k \quad \forall k \in K, \forall i \in N \cup \{n+1\} \quad (10)$$

$$L_0^k = 0 \quad \forall k \in K \quad (11)$$

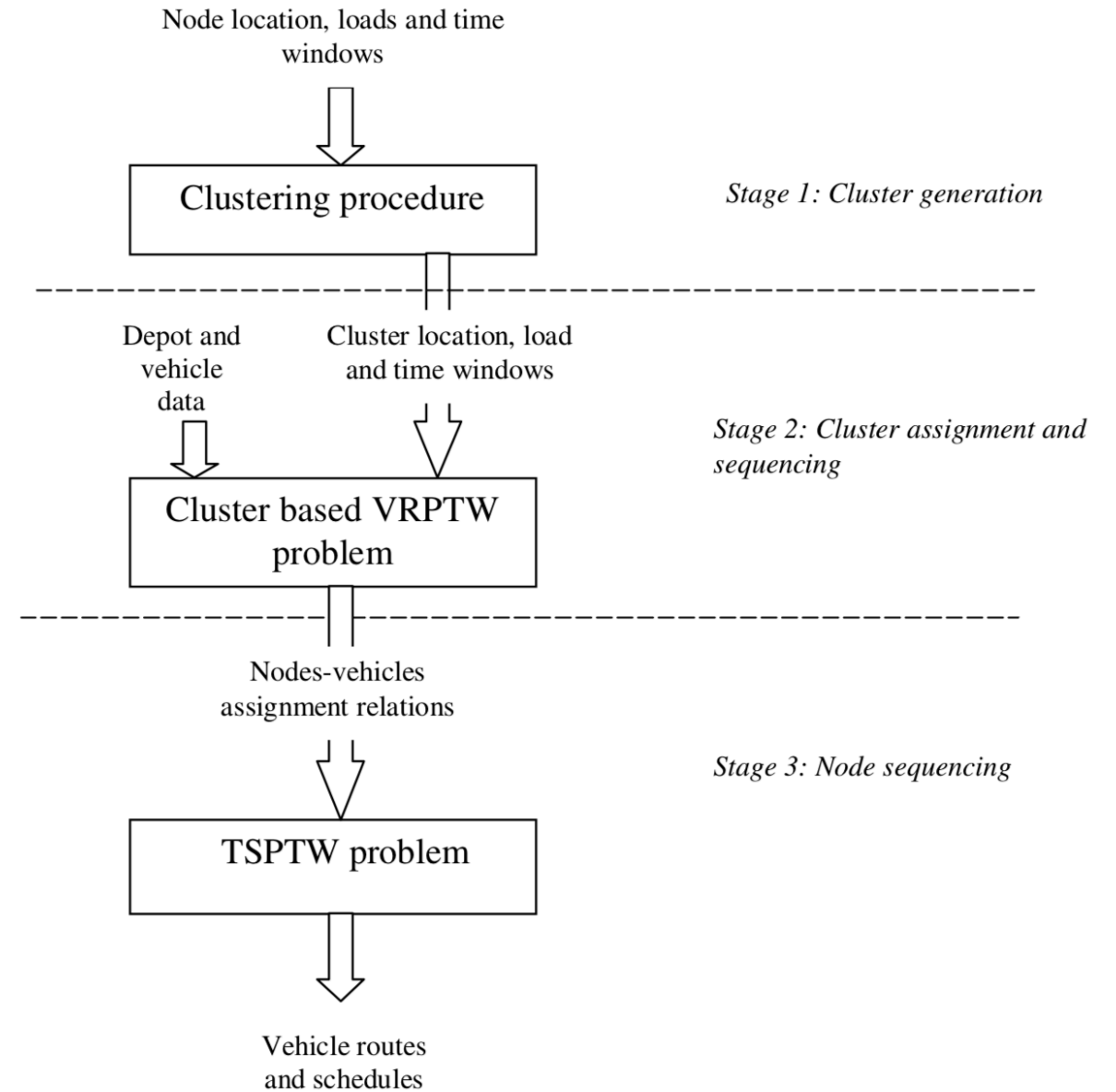
$$X_{ij}^k \geq 0 \quad \forall k \in K, (i,j) \in A \quad (12)$$

$$X_{ij}^k \in \{0, 1\} \quad \forall k \in K, (i,j) \in A \quad (13)$$

Clustering

- For a large-scale VRPTW problem, it is not realistic to apply pure optimization methods directly
- So, we can integrate a heuristic clustering procedure into the optimization framework
- Cluster first, route second
- Clusters of nodes are first defined, then such clusters are assigned to vehicles and sequenced on the related tours and finally the routing and scheduling for each individual tour in terms of the original nodes is separately found

Schematic of the VRPTW Clustering Approach



Stage 1

- By replacing nodes by clusters, stage 1 massively reduces the computational burden of the subsequent stages
- The cargo of one cluster can be assigned to a single vehicle
- There exists a route connecting the nodes on the cluster that satisfies all the time window constraints
- The vehicle waiting time because of early arrivals at pick-up/delivery points must be kept as small as possible

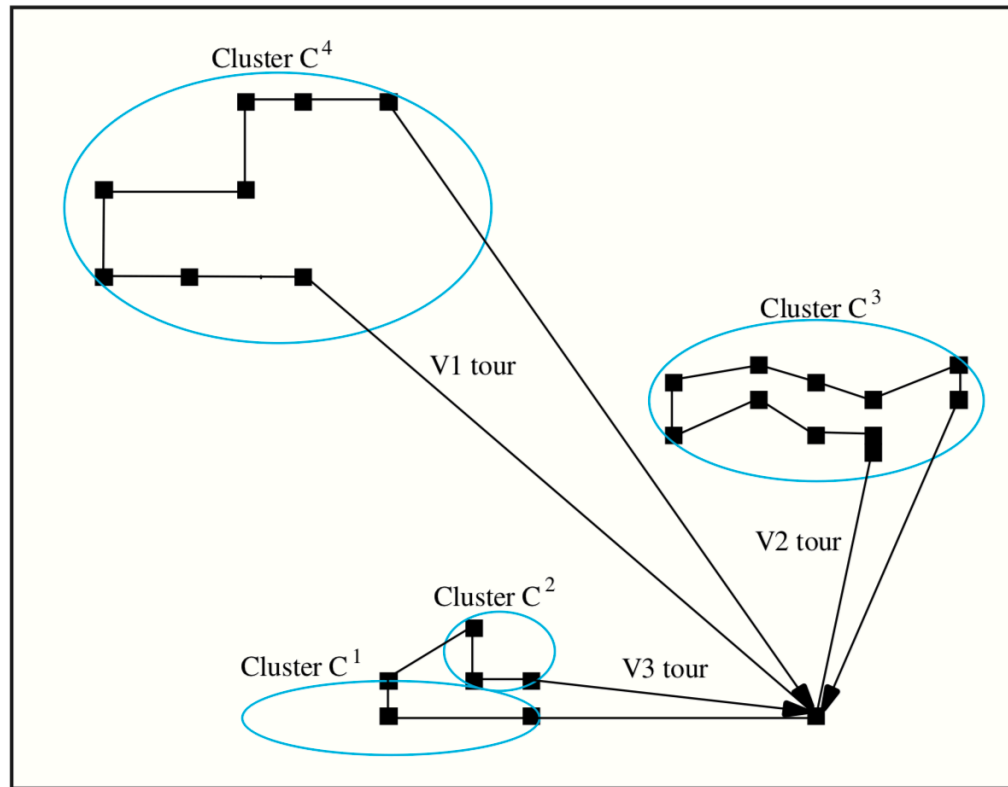
Stage 2

- Assignment of customer nodes to vehicles
- Discovery of a near-optimal set of cluster-based tours
- The sequence of clusters on the same tour that indirectly provides a partial arrangement of the customer sites visited by the same vehicle

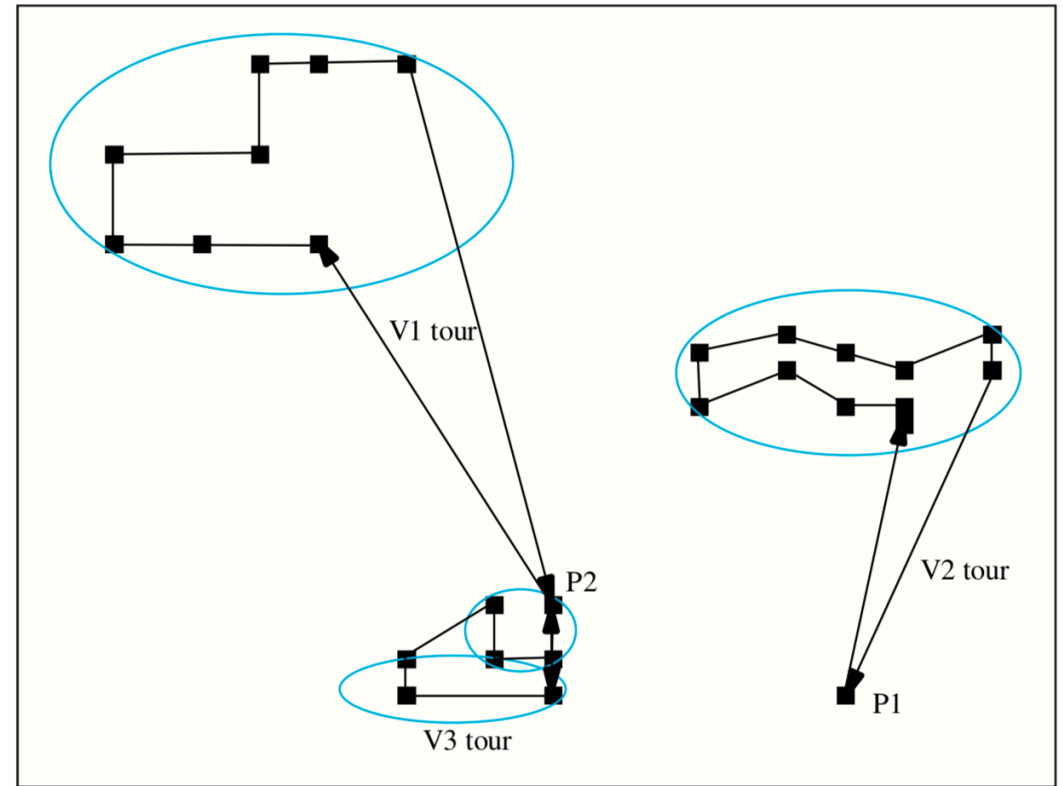
Stage 3

- Ordering nodes within clusters and scheduling the service start times at the customer locations for every tour is the aim of Stage 3
- For the nodes in the cluster linked to the same tour, a TSP formulation for each single-tour scheduling problem can be derived

Examples

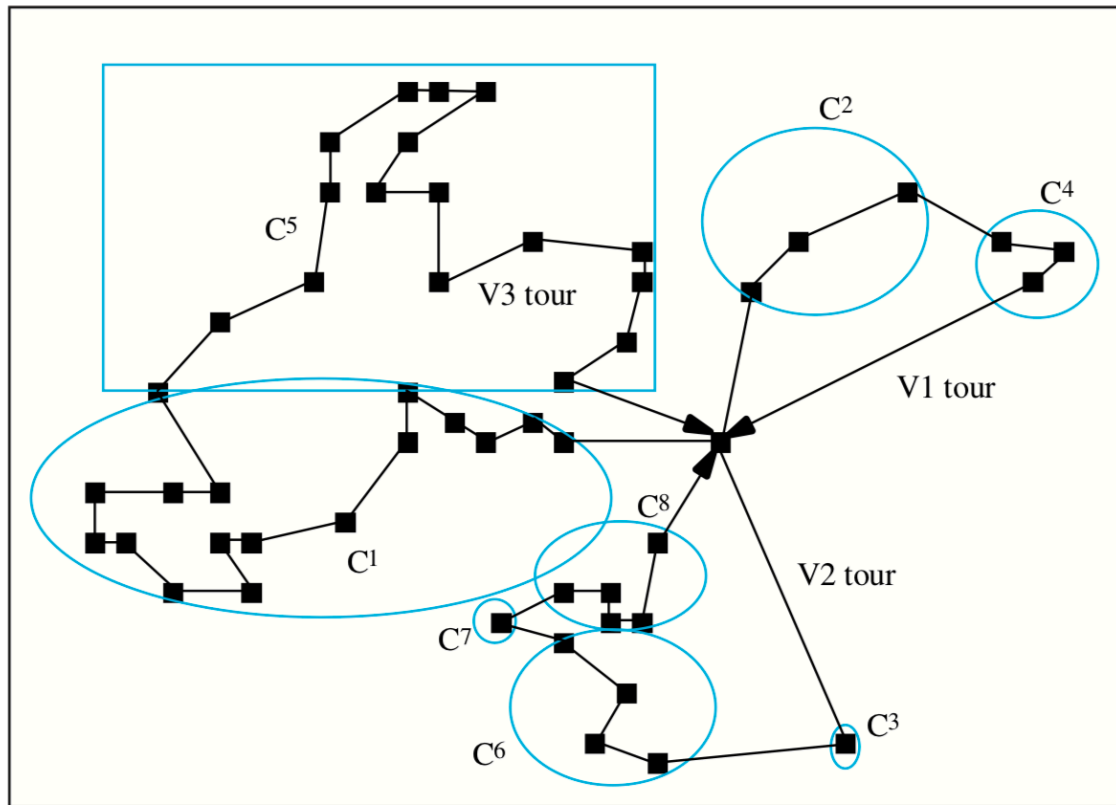


25 nodes

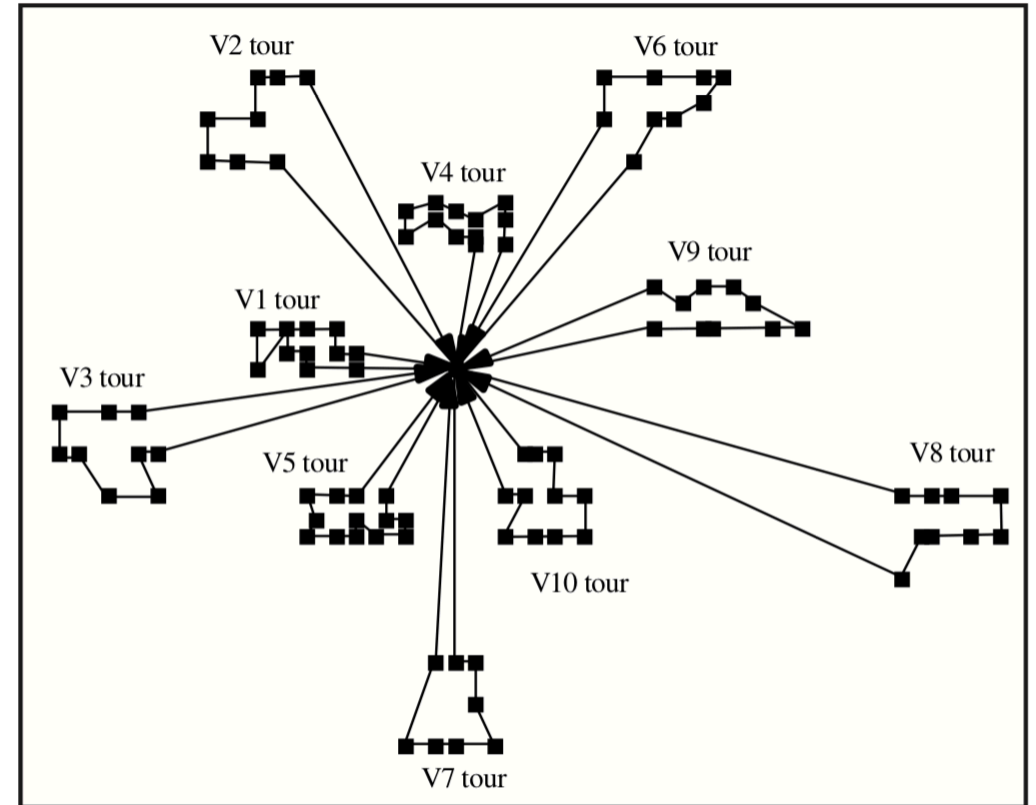


25 nodes, 2 depot

Examples



50 nodes



100 nodes

References

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Thank you!