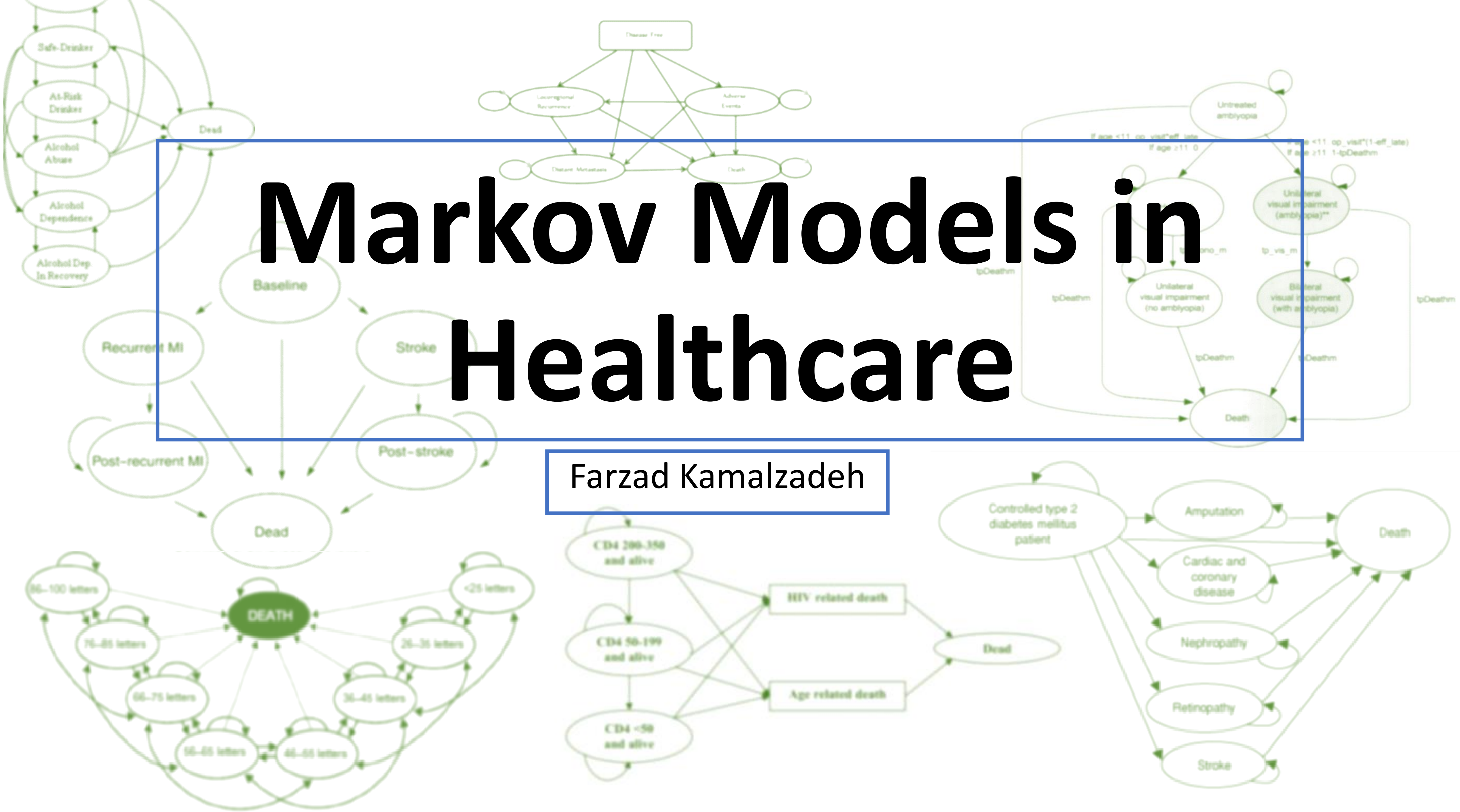
Markov Models in Healthcare

Farzad Kamalzadeh



Health Care Problem

- Chronic diseases: A **chronic disease** is a human health condition or disease that is persistent or otherwise long-lasting in its effects or a disease that comes with time.
- *Chronic:* the course of the disease lasts for more than 3 months.
- Common chronic diseases include:
  - arthritis
  - Asthma
  - Cancer
  - heart failure
  - diabetes
  - hepatitis C
  - HIV/AIDS

Epidemiology

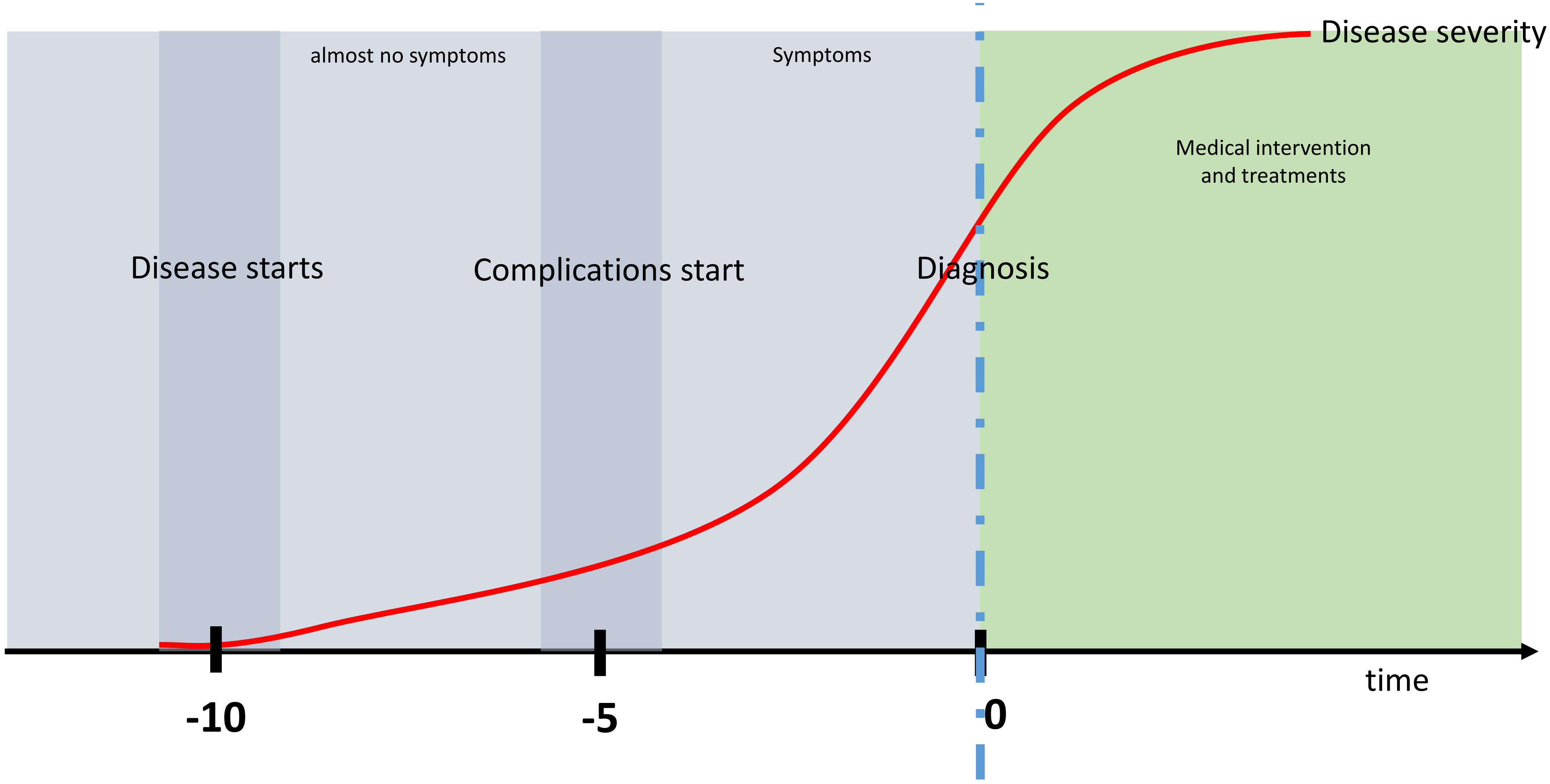
- Chronic diseases constitute a major cause of mortality:
  - WHO: **38 million deaths a year** to non-communicable diseases
  - United States: **25% of adults** have at least two chronic conditions
  - 1 in 2 Americans (**133 million**) has at least one chronic medical condition
  - **61%** of all **deaths** among people older than 65 in the population

- Diabetes:
  - 7th leading cause of death in the US
  - Leading cause of many complications such as kidney failure, non-traumatic lower limb amputations, blindness
  - Major cause of heart disease

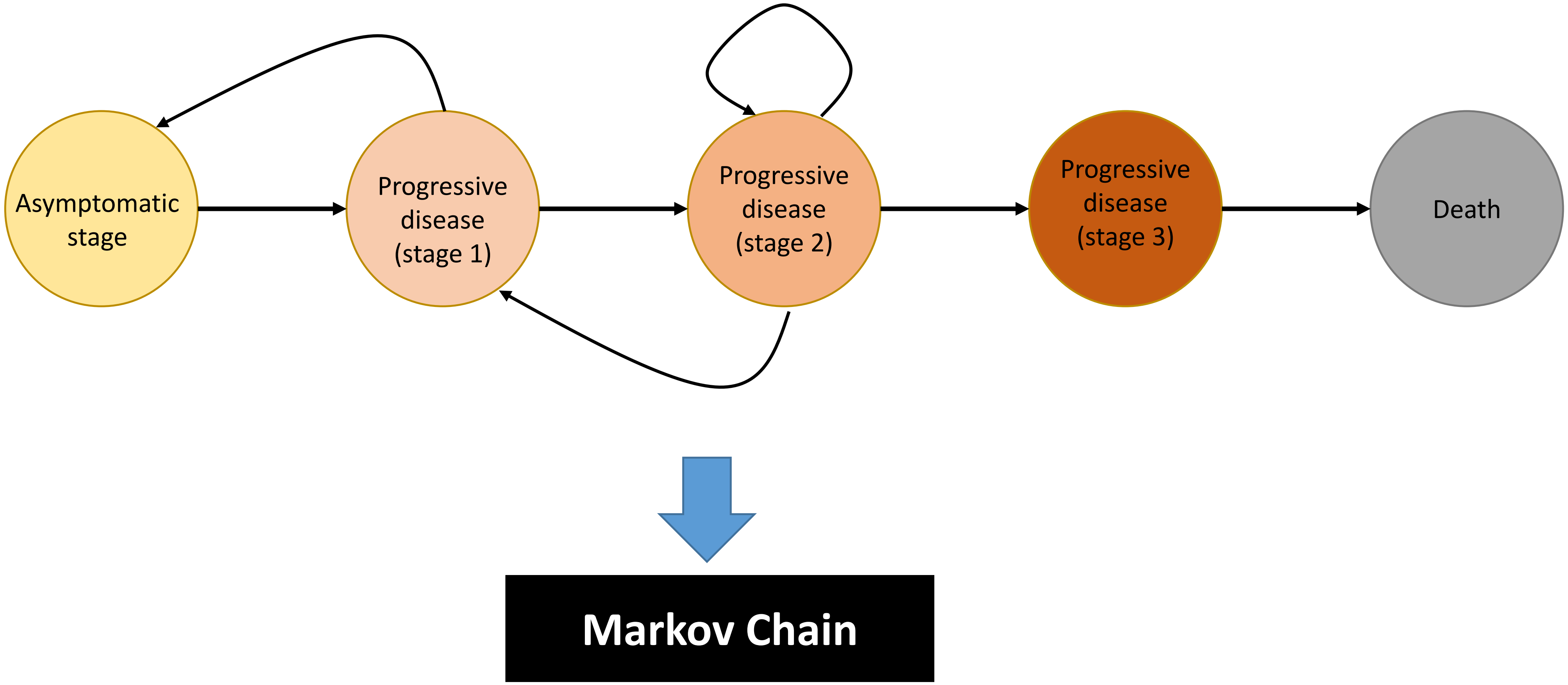
Economic impact

- Chronic diseases constitute a major section of medical care spending: (direct costs)
  - **75%** of the **$2 trillion** spent annually in US medical care ($1.5 trillion)
  - Diabetes: $1 in $3 Medicare expenditure
- (indirect costs)
  - limitations in daily activities
  - loss in productivity
  - loss of days of work
- Diabetes: $322 billion per year

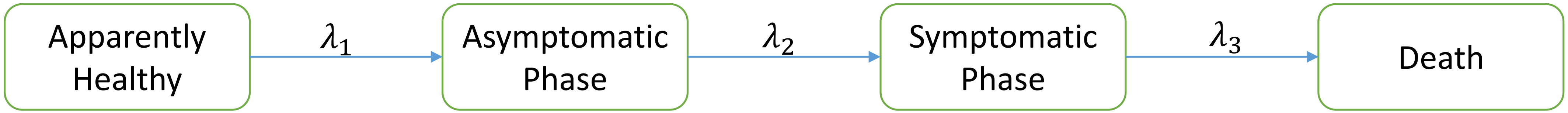
Nature of Chronic Diseases

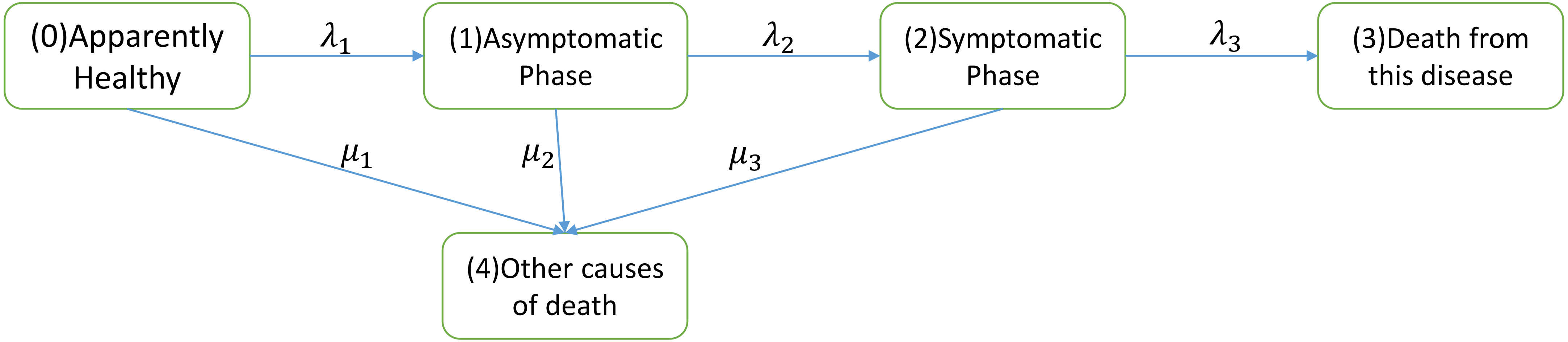


Natural Disease Progression



Markov Models





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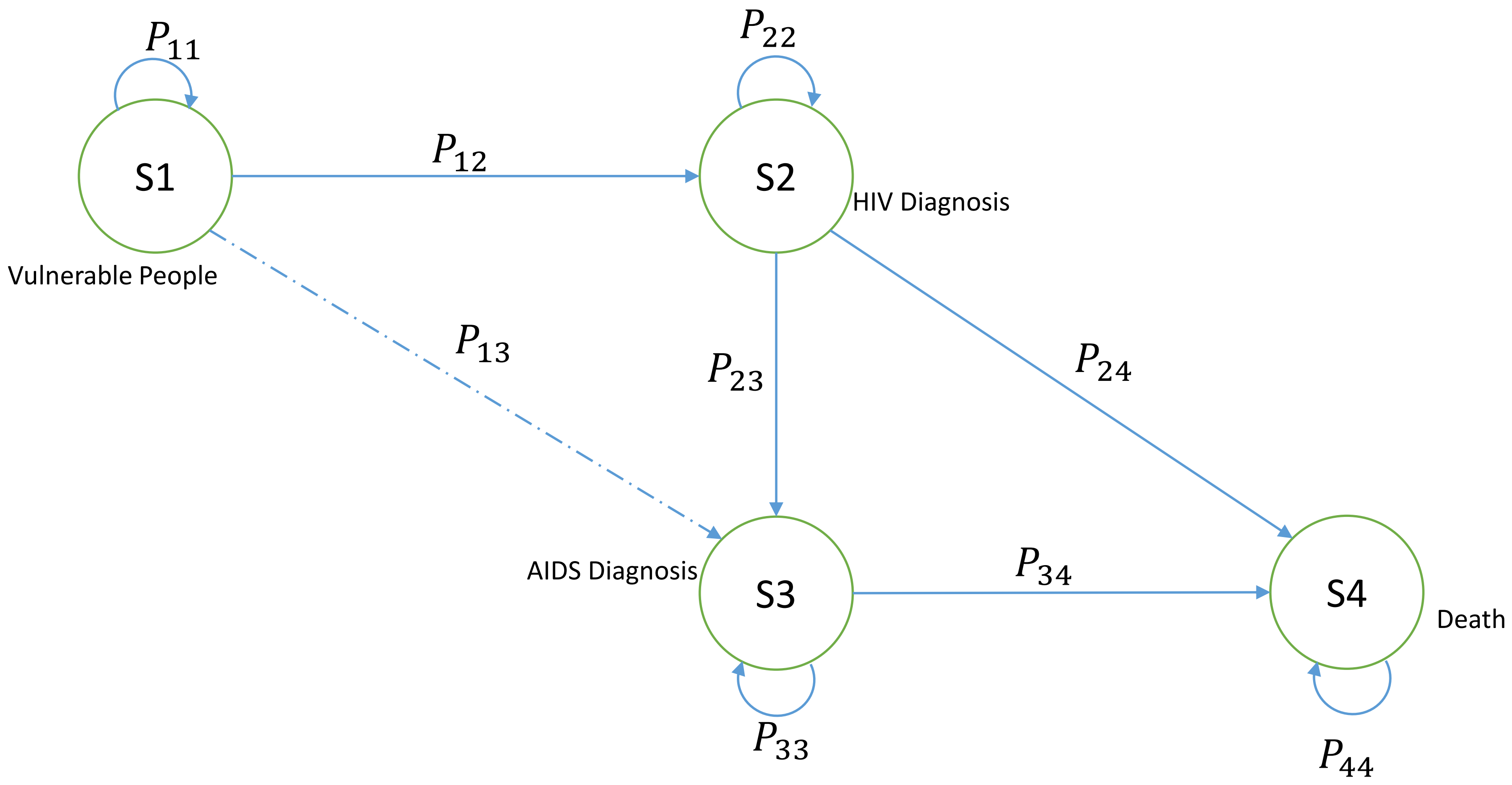

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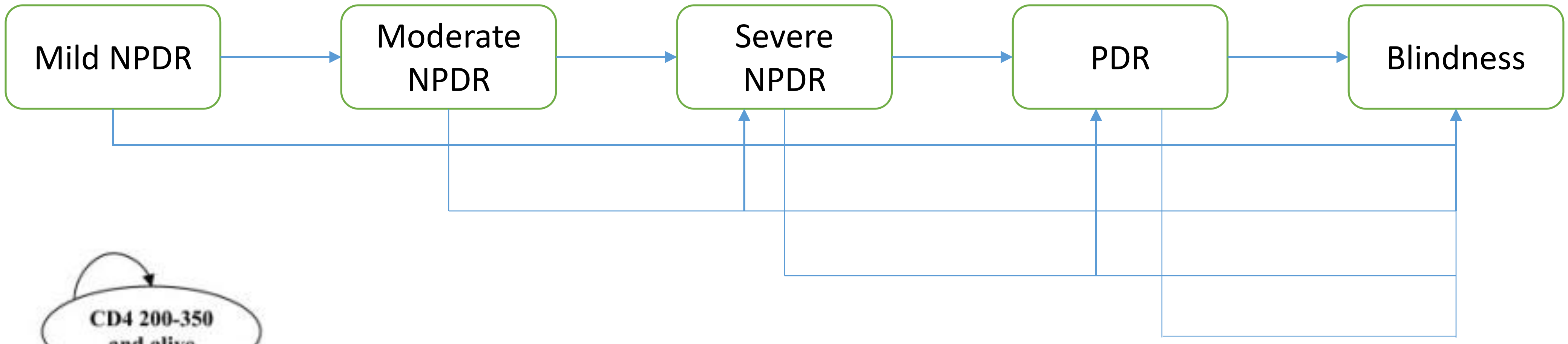

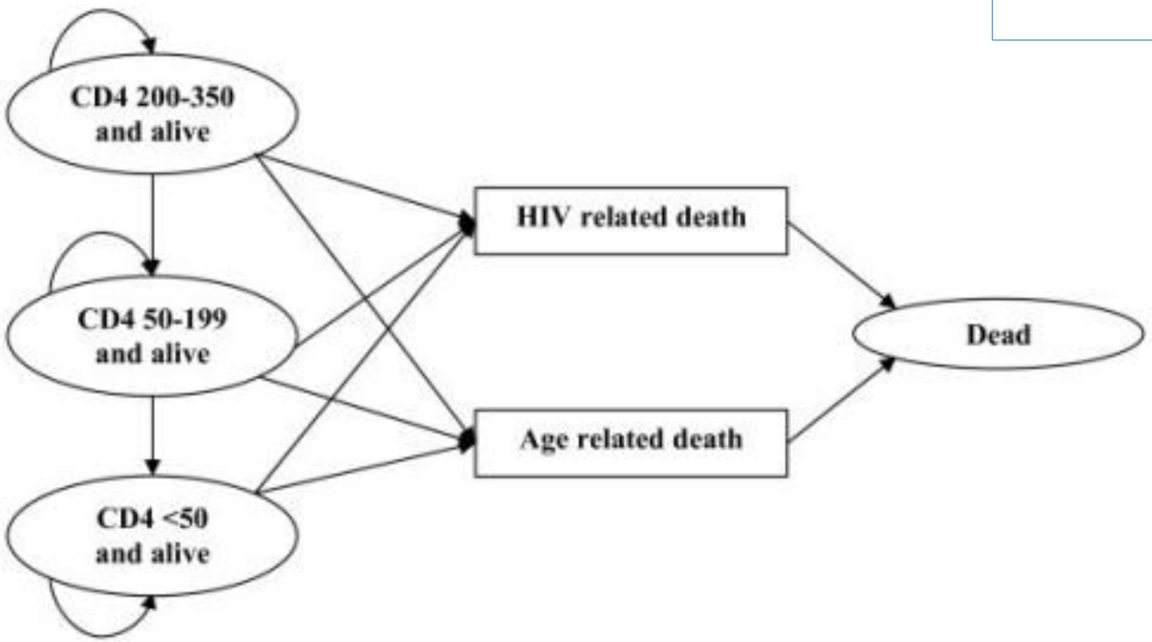

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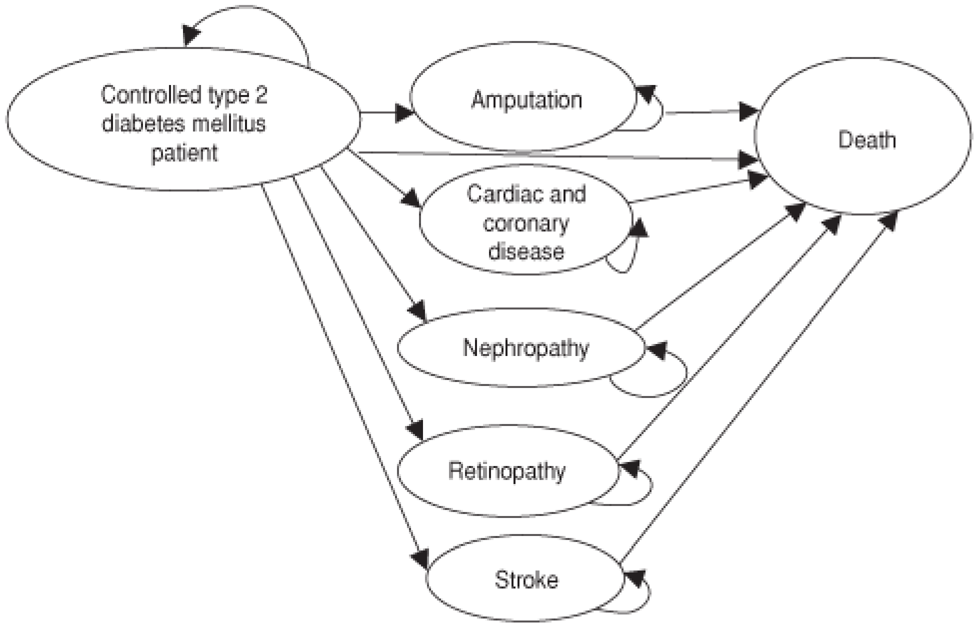

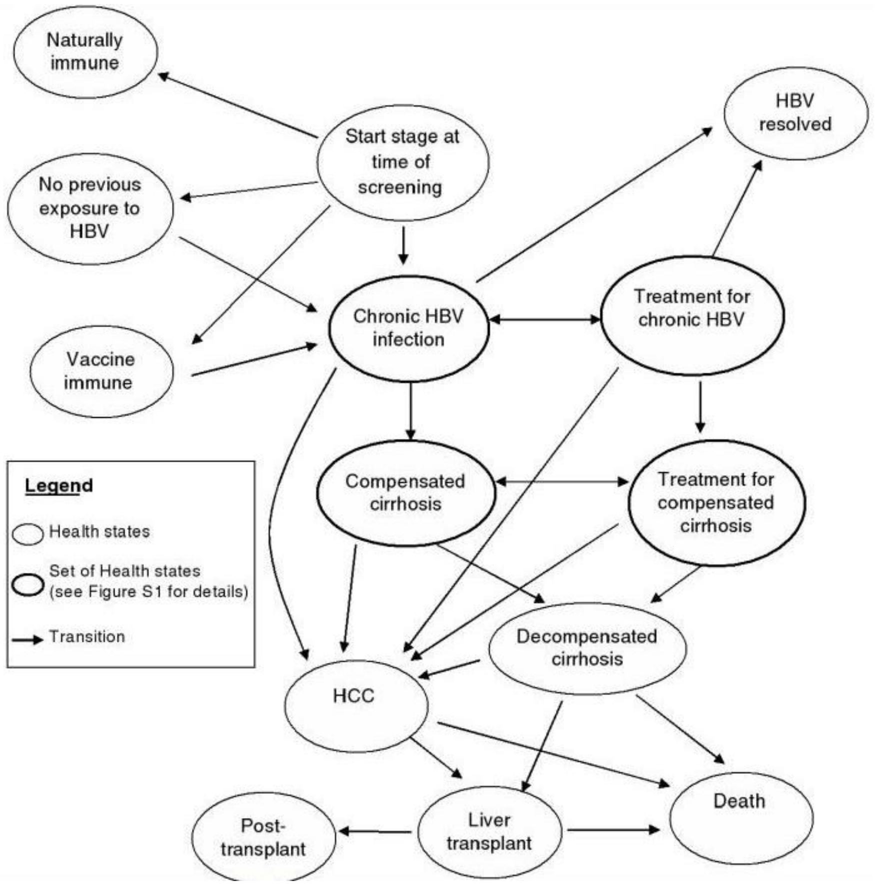

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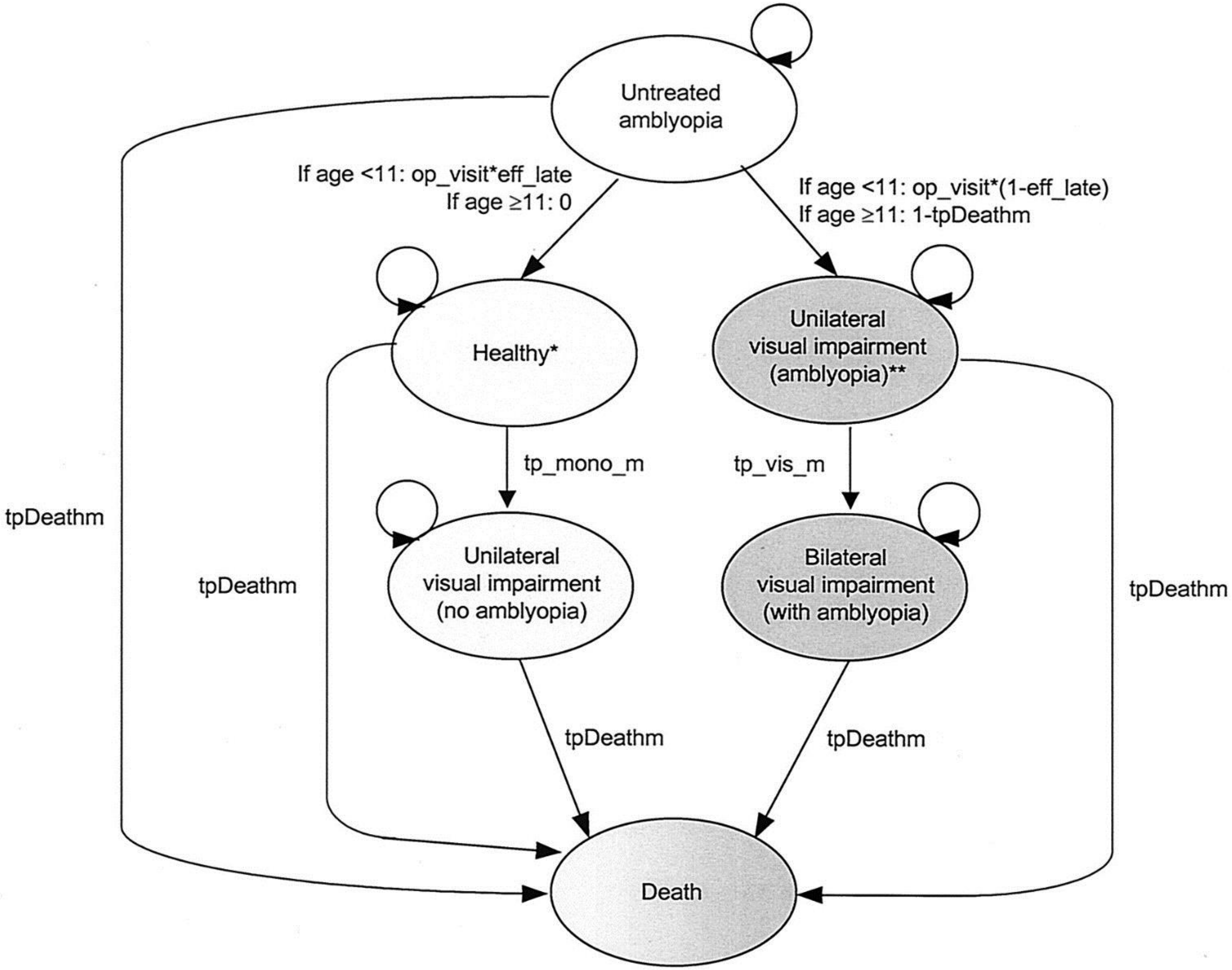

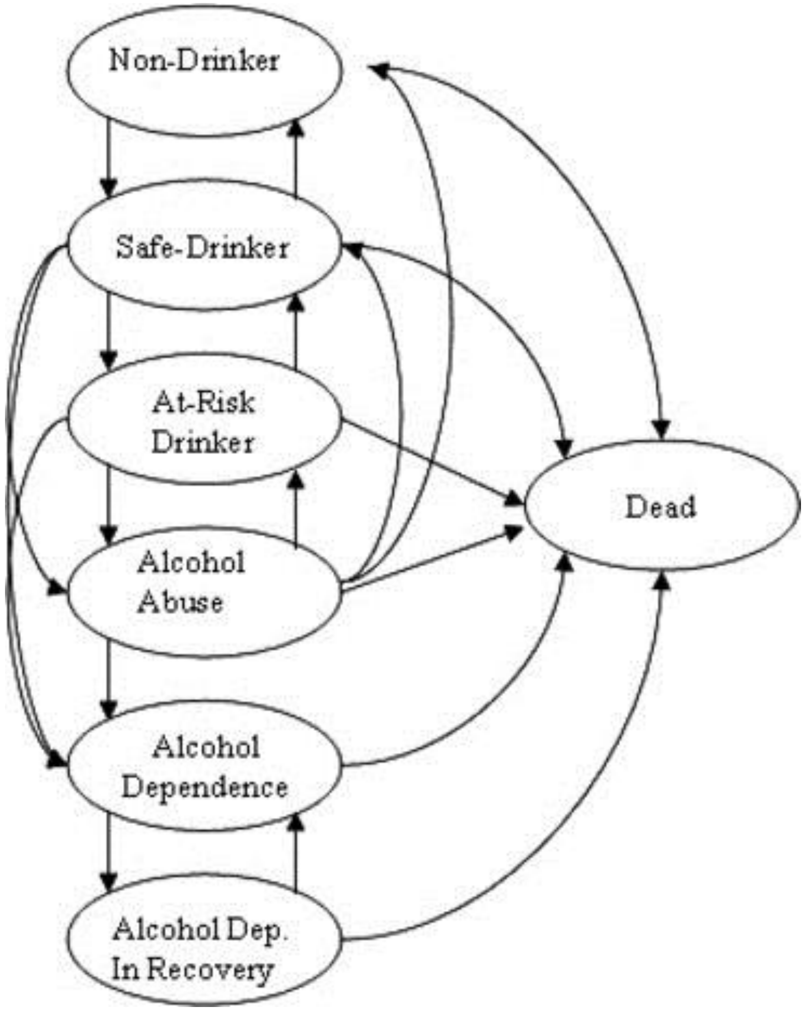

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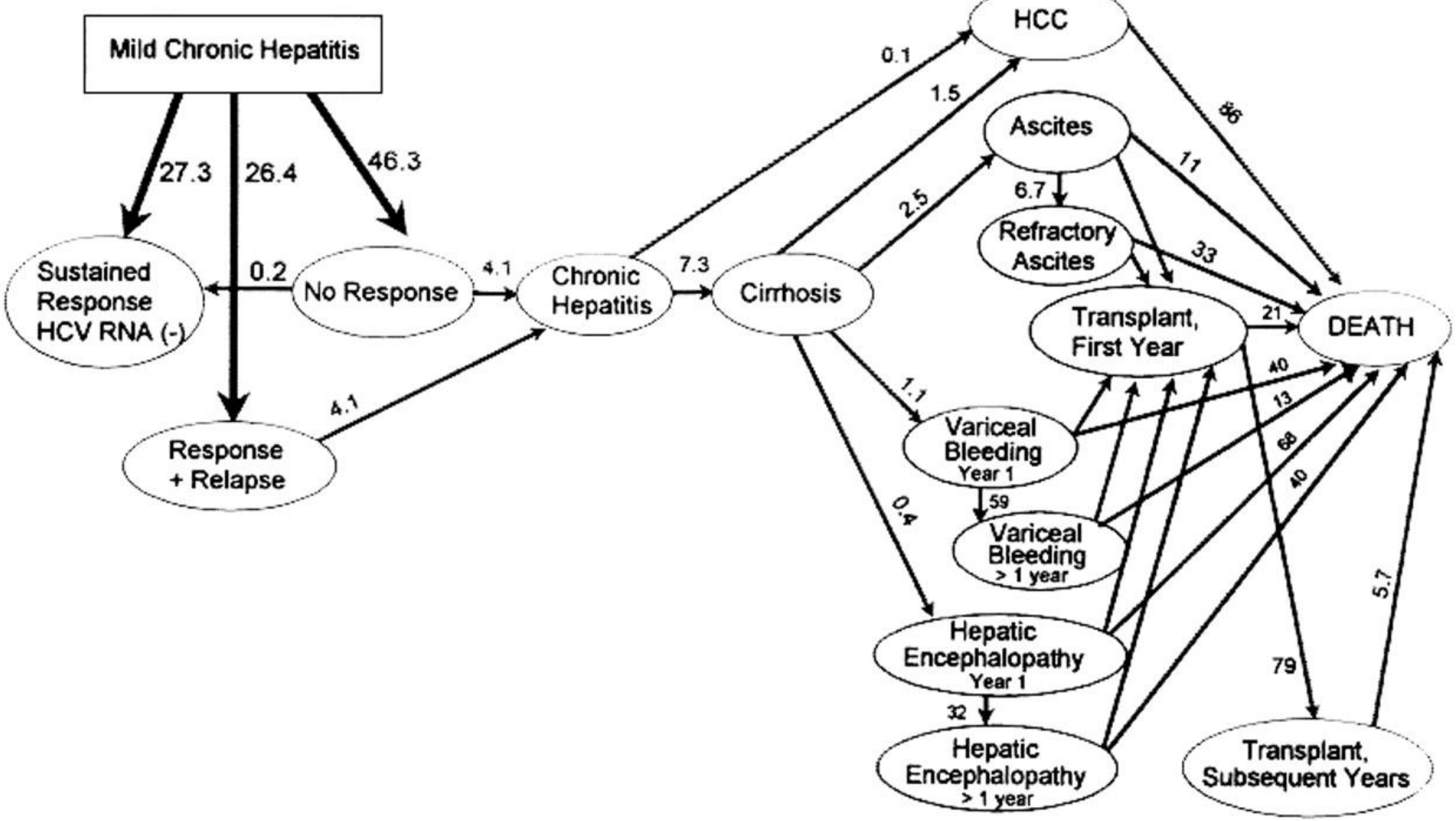

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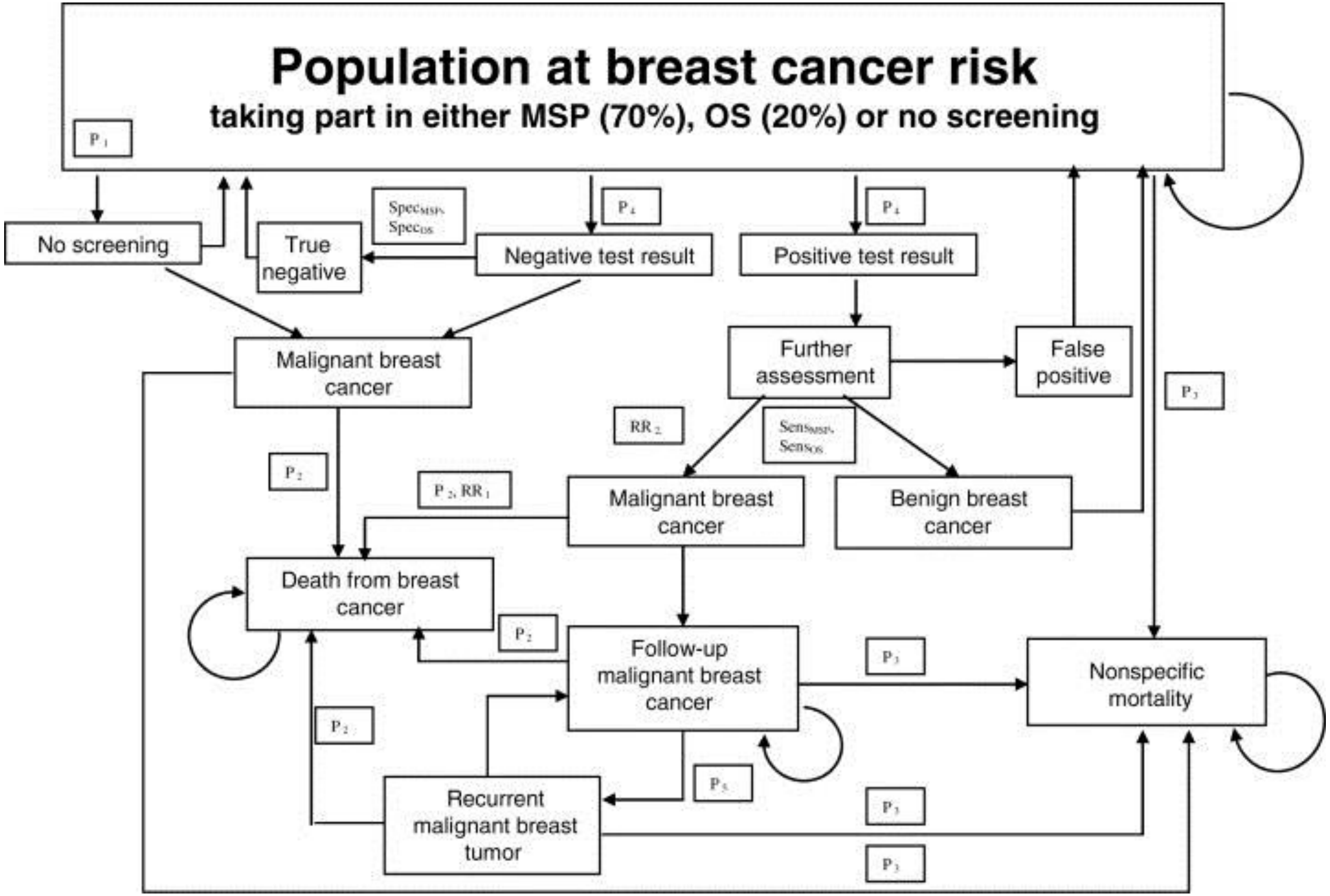
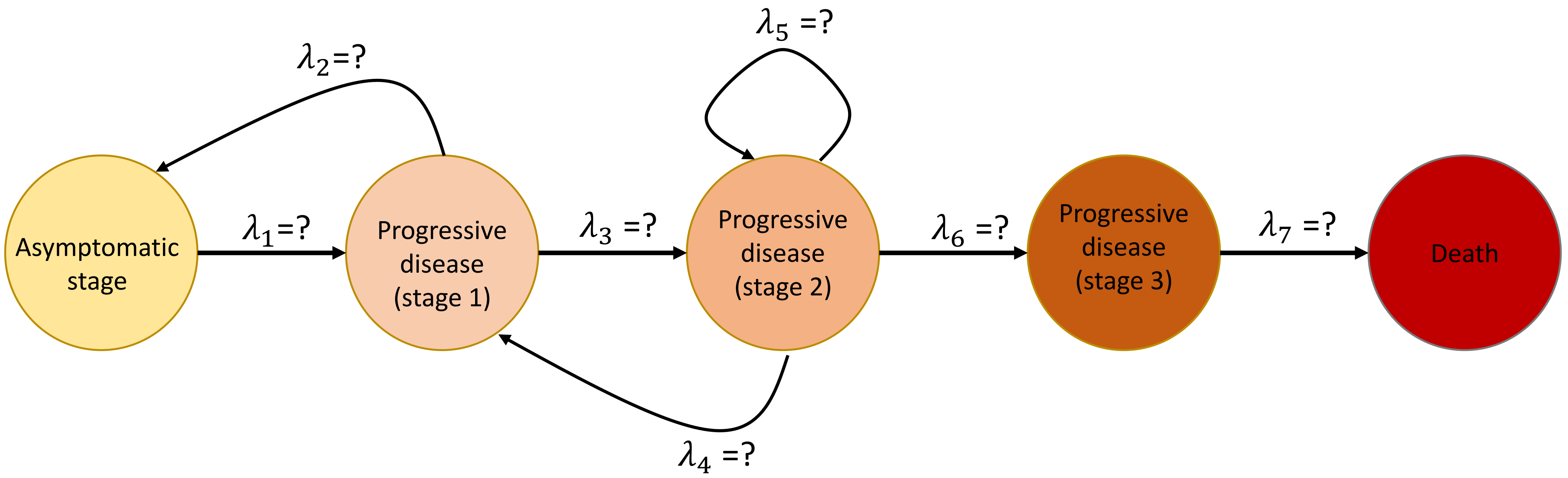
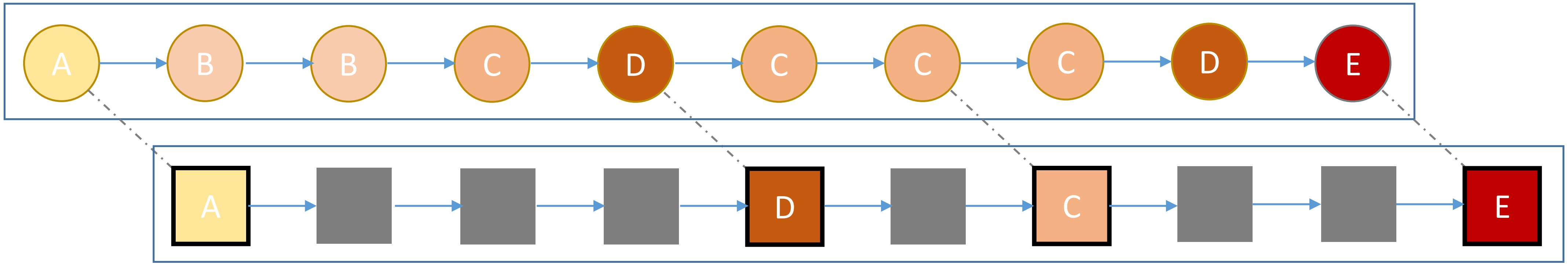


Markov Chains

Data Implementation

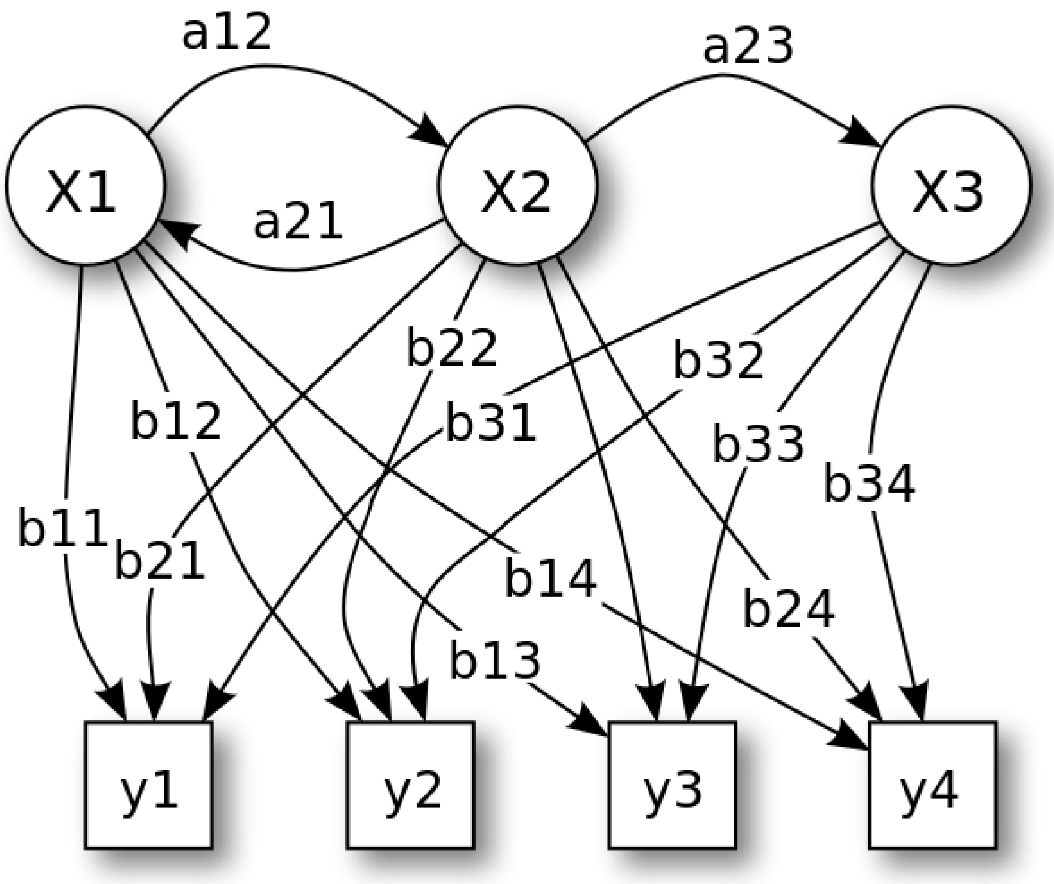


Data Implementation



	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=9	t=10
Patient 1	A				D		C			E
Patient 2	C		D	D		E				
Patient 3	A					B	A		A	
Patient 4		B		C						C
Patient 5	C		C					E		
Patient 6		D					D			E
Patient 7			B			C			B	

Hidden Markov Models



N	$=$	number of states
T	$=$	number of observations
$\theta_{i=1\ldots N}$	$=$	emission parameter for an observation associated with state i
$\phi_{i=1\ldots N, j=1\ldots N}$	$=$	probability of transition from state i to state j
$\boldsymbol{\phi}_{i=1\ldots N}$	$=$	N-dimensional vector,
$x_{t=1\ldots T}$	$=$	(hidden) state at time t
$y_{t=1\ldots T}$	$=$	observation at time t
$F(y\|\theta)$	$=$	probability distribution of an observation, parametrized on θ
$x_{t=2\ldots T}$	$\sim$	$\text{Categorical}(\boldsymbol{\phi}_{x_{t-1}})$
$y_{t=1\ldots T}$	$\sim$	$F(\theta_{x_t})$

Hidden Markov Models - Learning

- The parameter learning task in HMMs: given an output sequence or a set of such sequences ===> the best set of state transition probabilities.
- The task is usually to derive the maximum likelihood estimate of the parameters of the HMM given the set of output sequences
- local maximum likelihood can be derived efficiently using the Baum–Welch algorithm

Baum–Welch algorithm

$\lambda = (A, B, \pi)$

for each sequence

while desired level of convergence not acquired

for t= 1 to T

for i in S

$$\alpha_i(t) = P(Y_1 = y_1, Y_2 = y_2, \ldots, Y_t = y_t | X_t = i, \lambda)$$

the probability of seeing the $Y_1 = y_1, Y_2 = y_2, \ldots, Y_t = y_t$ and being in state i at time t

$$\beta_i(t) = P(Y_{t+1} = y_{t+1}, Y_{t+2} = y_{t+2}, \ldots, Y_T = y_T | X_t = i, \lambda)$$

the probability of the ending partial sequence $Y_{t+1} = y_{t+1}, Y_{t+2} = y_{t+2}, \ldots, Y_T = y_T$ given starting state i at time t

$$\gamma_i(t) = P(X_t = i | Y, \lambda) = \frac{\alpha_i(t).\beta_i(t)}{\sum_{j=1}^{N} \alpha_j(t).\beta_j(t)}$$

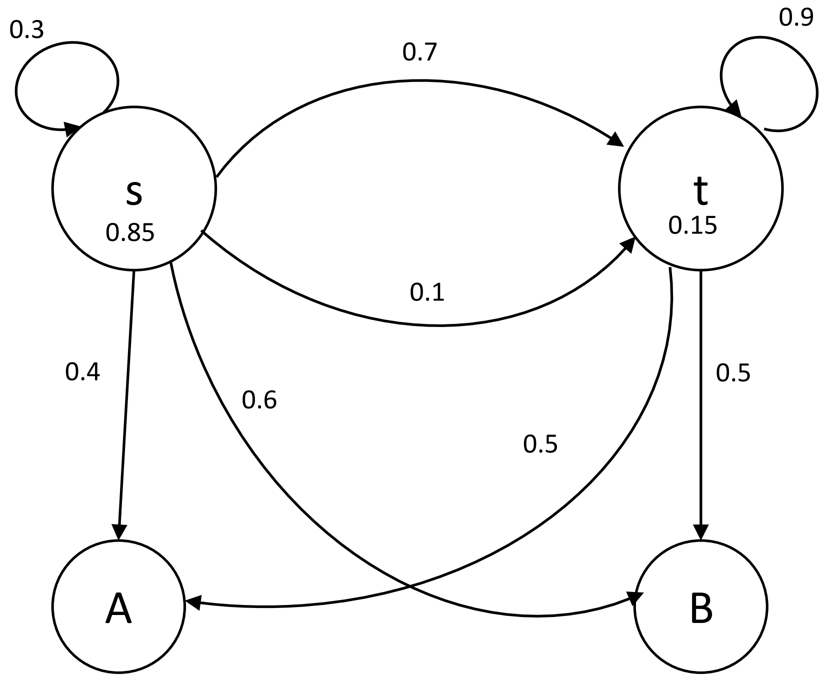
the probability of being in state i at time t given the observed sequence Y and the parameters λ

$$\delta_{ij}(t) = P(X_t = i, X_{t+1} = \mathrm{j} | Y, \lambda) = \frac{\alpha_i(t) a_{ij}.\beta_i(t+1) b_j(y_{t+1})}{\sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i(t) a_{ij}.\beta_i(t+1) b_j(y_{t+1})}$$

the probability of being in state i and j at times t and t+1 respectively given the observed sequence Y and parameters λ

$$update: \quad \pi_i = \gamma_i(1) \quad a_{ij} = \frac{\sum_{t=1}^{T-1} \delta_{ij}(t)}{\sum_{t=1}^{T-1} \gamma_i(t)} \quad b_i(v_k) = \frac{\sum_{t=1}^{T} 1_{y_t = v_k} \gamma_i(t)}{\sum_{t=1}^{T} \gamma_i(t)}$$

Baum–Welch algorithm



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