World Changers Shaped Here SMU

Bayesian Networks

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Bayesian Networks

Belief networks

Probabilistic graphical models

- Probability
- Graphs

Probability

Formula:

$$p(x, y) = p(x|y)p(y) = p(y|x)p(x)$$

Which is derived from Bayes' Rule

$$p(x|y) = \frac{p(x, y)}{p(y)} = \frac{p(y|x)p(x)}{p(y)}$$

Graphs

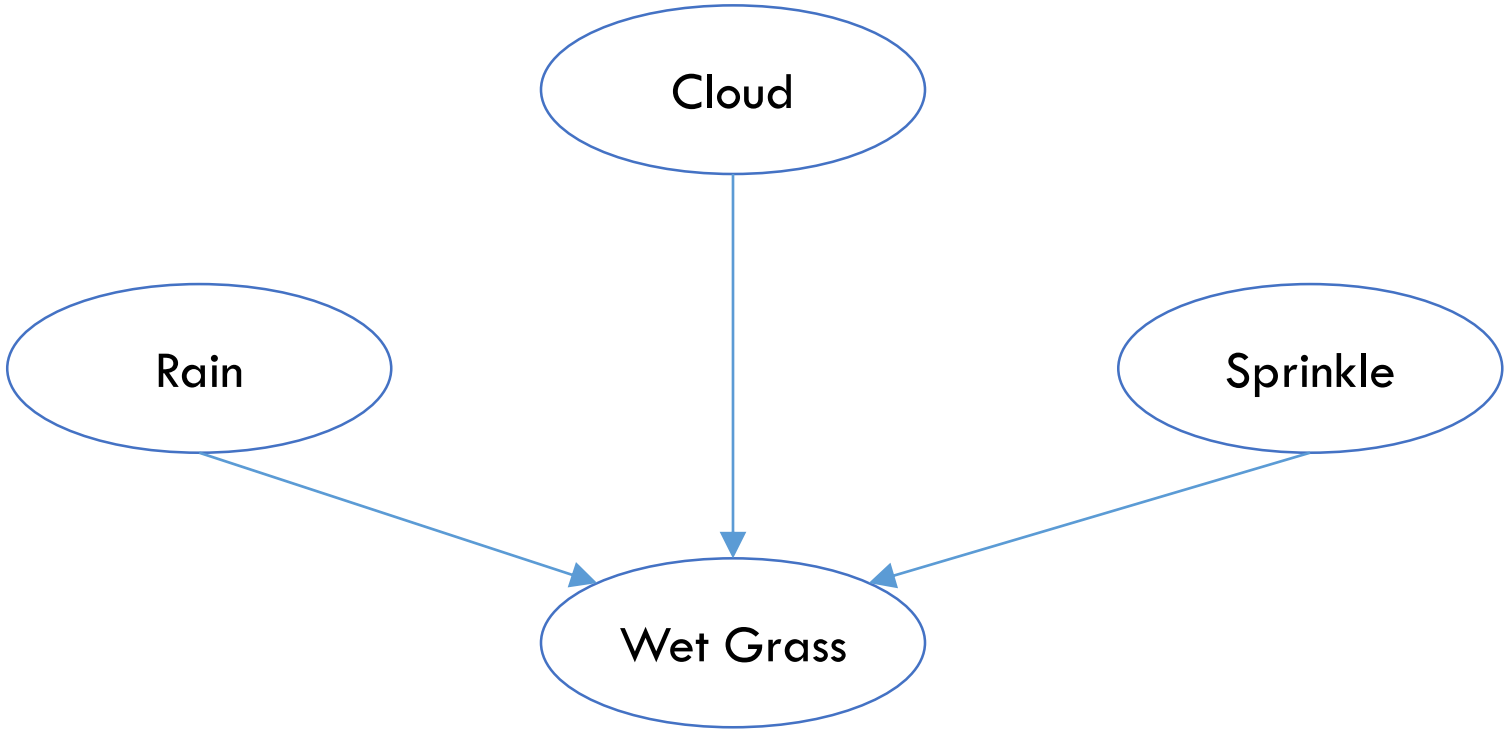
Graph

Representation of a set of objects where some pairs of objects are connected by edges

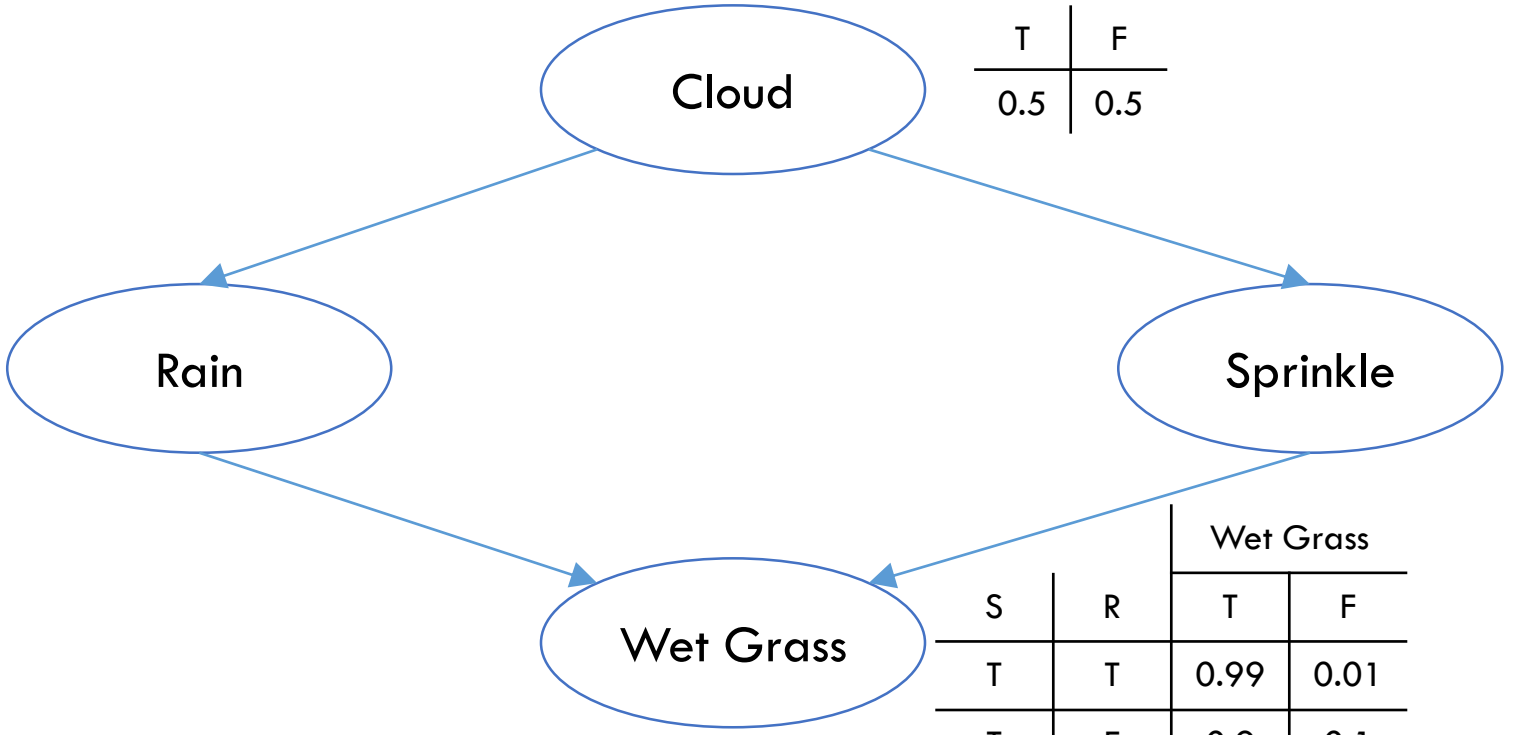
Directed Acyclic Graph

A graph where the edges have a direction associated with them and no directed cycles

Naïve Bayesian Network



Non-naïve Bayesian Network



T	F
0.5	0.5

	Rain	
Cloud	T	F
T	0.8	0.2
F	0.2	0.8

	Sprinkle	
Cloud	T	F
T	0.1	0.9
F	0.5	0.5

		Wet Grass	
S	R	T	F
T	T	0.99	0.01
T	F	0.9	0.1
F	T	0.9	0.1
F	F	0	1

Joint Probability Distribution

Cloud	Rain	Sprinkle	Wet Grass	p(w,s,r,c)
1	1	1	1	?
1	1	1	0	?
1	1	0	1	?
1	1	0	0	?
1	0	1	1	?
1	0	1	0	?
1	0	0	1	?
1	0	0	0	?

Cloud	Rain	Sprinkle	Wet Grass	p(w,s,r,c)
0	1	1	1	?
0	1	1	0	?
0	1	0	1	?
0	1	0	0	?
0	0	1	1	?
0	0	1	0	?
0	0	0	1	?
0	0	0	0	?

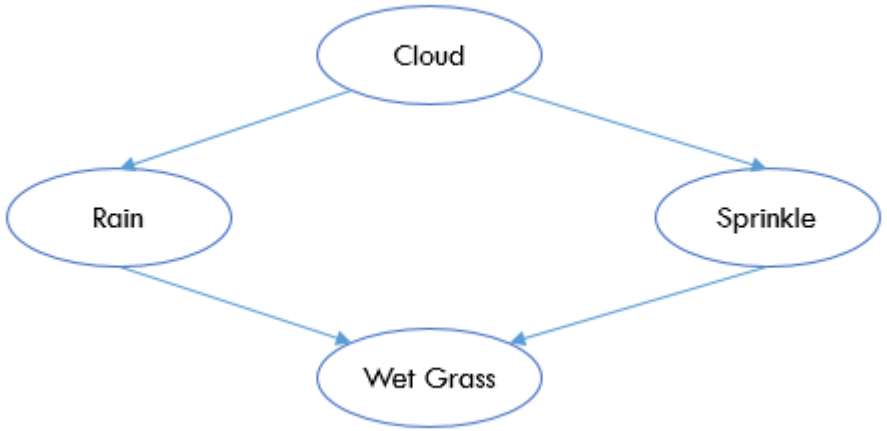
$$p(w,s,r,c) = p(w|s,r,c)\, p(s|r,c)\, p(r|c)\, p(c)$$

Conditional Independence

Definition: X is conditionally independent of Y given Z if the probability distribution governing X is independent of the value of Y given the value of Z

$$p(x|y,z) = p(x|z)$$

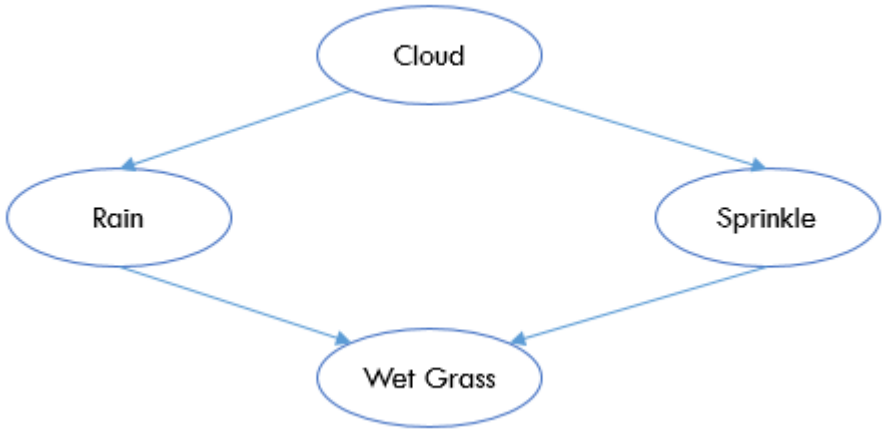
Example: Wet Grass is conditionally independent of Cloud given rain



Conditional Independence

$$p(w, s, r, c) = p(w|s, r, c)\, p(s|r, c)\, p(r|c)\, p(c)$$

$$p(w, s, r, c) = p(w|s, r)\, p(s|c)\, p(r|c)\, p(c)$$



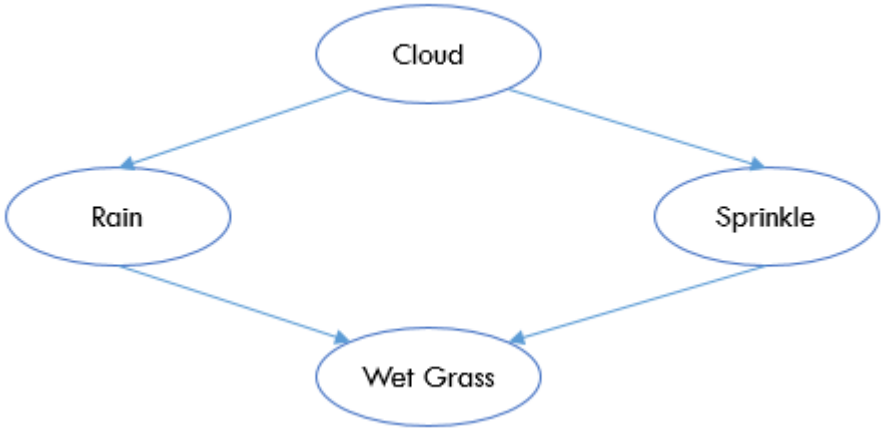
Example

$$p(w, s, r, c) = p(w|s, r)\, p(s|c)\, p(r|c)\, p(c)$$

$$p(T, T, T, T) = 0.99 * 0.1 * 0.8 * 0.5 = 0.0396$$

$$p(F, T, T, T) = 0.01 * 0.1 * 0.8 * 0.5 = 0.0004$$

Probabilistic Inference



Two possible causes

- Raining

$$p(r|w) = \frac{p(r,w)}{p(w)} = \frac{0.4581}{0.6471} = 0.708$$

- Sprinkle is on

$$p(s|w) = \frac{p(s,w)}{p(w)} = \frac{0.2781}{0.6471} = 0.43$$

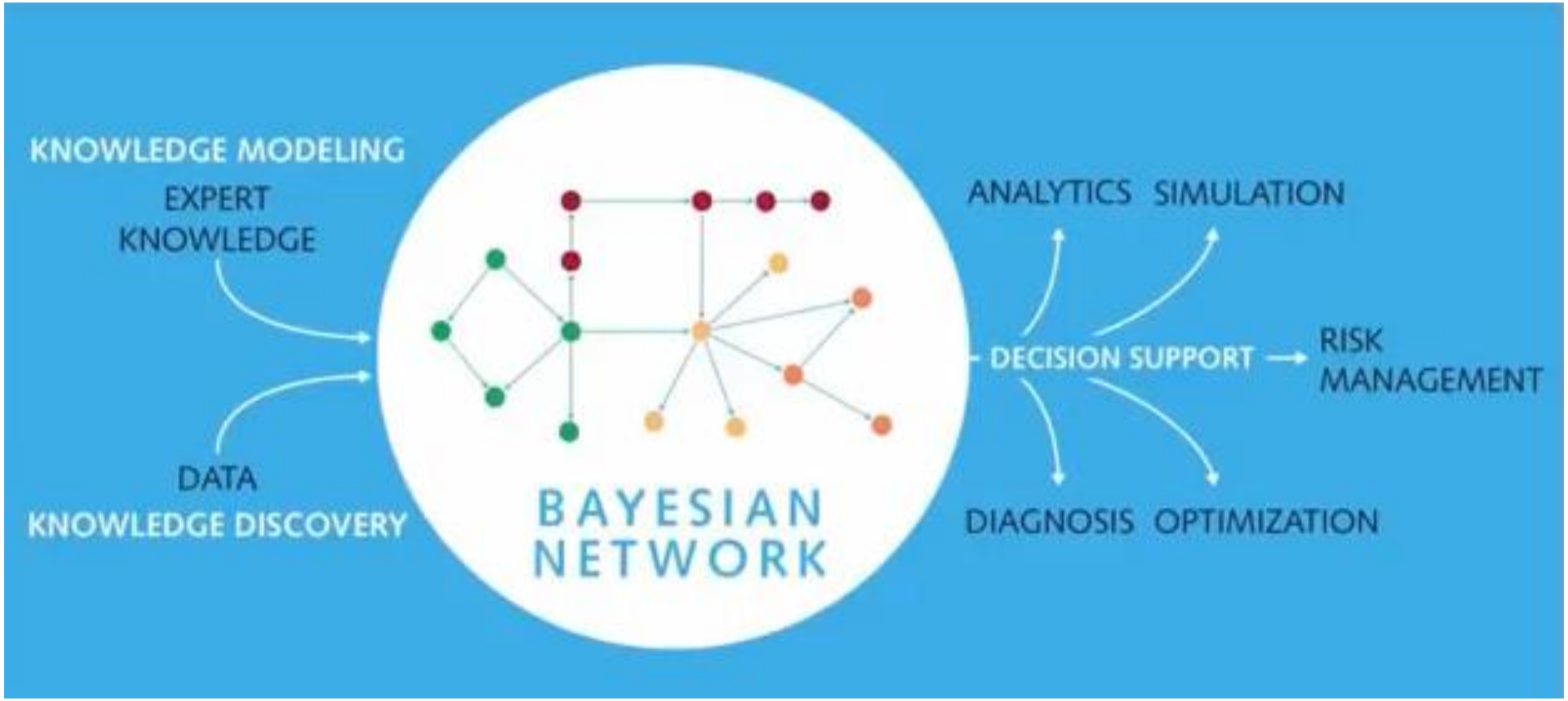
Bayesian Network: Step

1. Identify the goals of modeling
2. Identify many possible observation which relevant to the problem
3. Build a directed acyclic graph with Conditional Probability table
4. Compute the probability using formula

Summary

- Update the probabilities of variables whose state has not been observed given some set of new observations
- Automate the process of the above by combining qualitative and quantitative information

Summary



Bayesia: http://www.bayesia.us/

What is Next?

- Method to handle missing data
  - Monte-Carlo Methods: Accurate, but long computations
  - Gaussian Approximation
- Method to learn Structure of Network
  - Machine Learning

Bayesian Network in R

- Packages
  - bnlearn
    - Bayesian Network Structure Learning, Parameter learning and inference
  - gRain
    - Specify a network, Querying a network, and etc