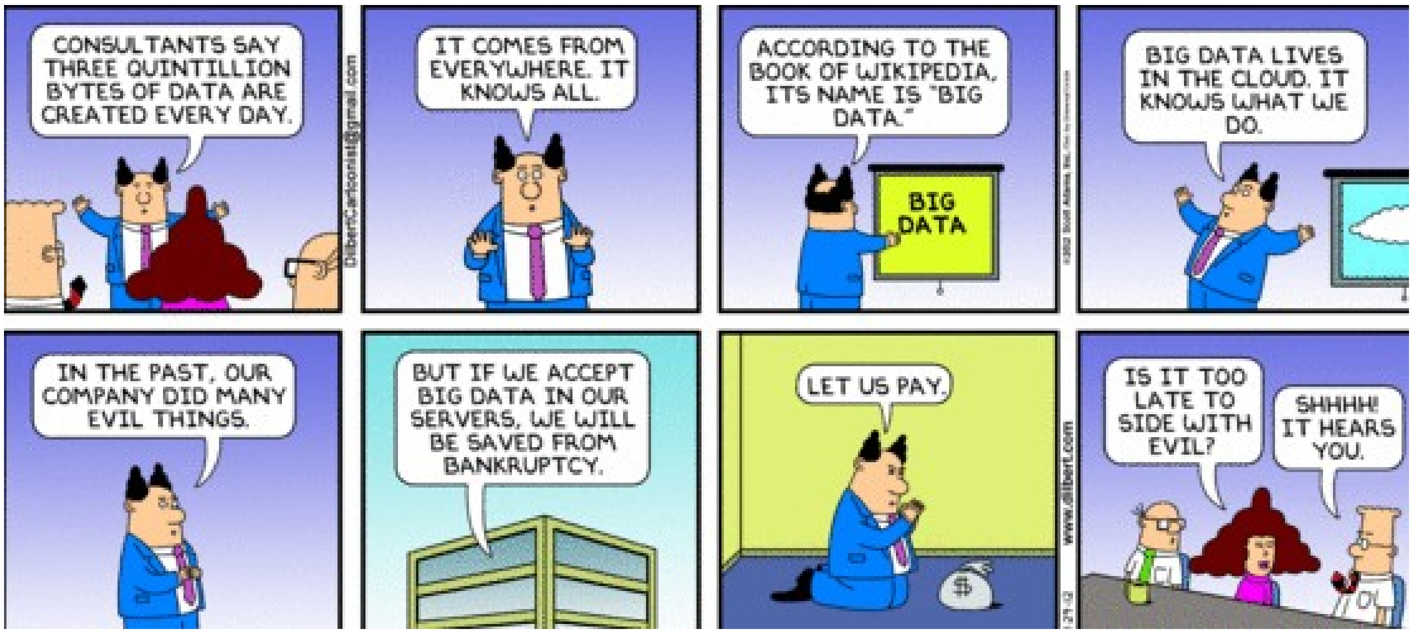
Association Analysis: Basic Concepts and Algorithms



Lecture Notes for Chapter 6

Slides by Tan, Steinbach, Kumar adapted by Michael Hahsler

Look for accompanying R code on the course web site.

Topics

- Definition
- Mining Frequent Itemsets (APRIORI)
- Concise Itemset Representation
- Alternative Methods to Find Frequent Itemsets
- Association Rule Generation
- Support Distribution
- Pattern Evaluation

Association Rule Mining

- Given a set of transactions, find rules that will predict the occurrence of an item based on the occurrences of other items in the transaction

Market-Basket transactions

TID	*Items*
1	**Bread, Milk**
2	**Bread, Diaper, Beer, Eggs**
3	**Milk, Diaper, Beer, Coke**
4	**Bread, Milk, Diaper, Beer**
5	**Bread, Milk, Diaper, Coke**

Example of Association Rules

{Diaper} → {Beer},
{Milk, Bread} → {Eggs,Coke},
{Beer, Bread} → {Milk},

Implication means co-occurrence, not causality!

Definition: Frequent Itemset

- **Itemset**
  - A collection of one or more items
    - Example: {Milk, Bread, Diaper}
  - k-itemset
    - An itemset that contains k items
- **Support count (σ)**
  - Frequency of occurrence of an itemset
  - E.g. σ({Milk, Bread,Diaper}) = 2
- **Support**
  - Fraction of transactions that contain an itemset
  - E.g. s({Milk, Bread, Diaper}) = σ({Milk, Bread,Diaper}) / |T| = 2/5
- **Frequent Itemset**
  - An itemset whose support is greater than or equal to a *minsup* threshold

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

$$s(X) = \frac{\sigma(X)}{|T|}$$

Definition: Association Rule

- **Association Rule**
  - An implication expression of the form $X \rightarrow Y$, where X and Y are itemsets
  - Example:
    {Milk, Diaper} → {Beer}

TID	*Items*
1	**Bread, Milk**
2	**Bread, Diaper, Beer, Eggs**
3	**Milk, Diaper, Beer, Coke**
4	**Bread, Milk, Diaper, Beer**
5	**Bread, Milk, Diaper, Coke**

- **Rule Evaluation Metrics**
  - Support (s)
    - Fraction of transactions that contain both X and Y
  - Confidence (c)
    - Measures how often items in Y appear in transactions that contain X

Example:

$$\{\text{Milk}, \text{Diaper}\} \Rightarrow \text{Beer}$$

$$s = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{|T|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{\sigma(\text{Milk}, \text{Diaper})} = \frac{2}{3} = 0.67$$

$$c(X \rightarrow Y) = \frac{\sigma(X \cup Y)}{\sigma(X)} = \frac{s(X \cup Y)}{s(X)}$$

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Association Rule Mining Task

- Given a set of transactions T, the goal of association rule mining is to find all rules having
  - support ≥ *minsup* threshold
  - confidence ≥ *minconf* threshold

- Brute-force approach:
  - List all possible association rules
  - Compute the support and confidence for each rule
  - Prune rules that fail the *minsup* and *minconf* thresholds

  ⇒ Computationally prohibitive!

Mining Association Rules

TID	*Items*
1	**Bread, Milk**
2	**Bread, Diaper, Beer, Eggs**
3	**Milk, Diaper, Beer, Coke**
4	**Bread, Milk, Diaper, Beer**
5	**Bread, Milk, Diaper, Coke**

Example of Rules:

{Milk,Diaper} → {Beer} (s=0.4, c=0.67)
{Milk,Beer} → {Diaper} (s=0.4, c=1.0)
{Diaper,Beer} → {Milk} (s=0.4, c=0.67)
{Beer} → {Milk,Diaper} (s=0.4, c=0.67)
{Diaper} → {Milk,Beer} (s=0.4, c=0.5)
{Milk} → {Diaper,Beer} (s=0.4, c=0.5)

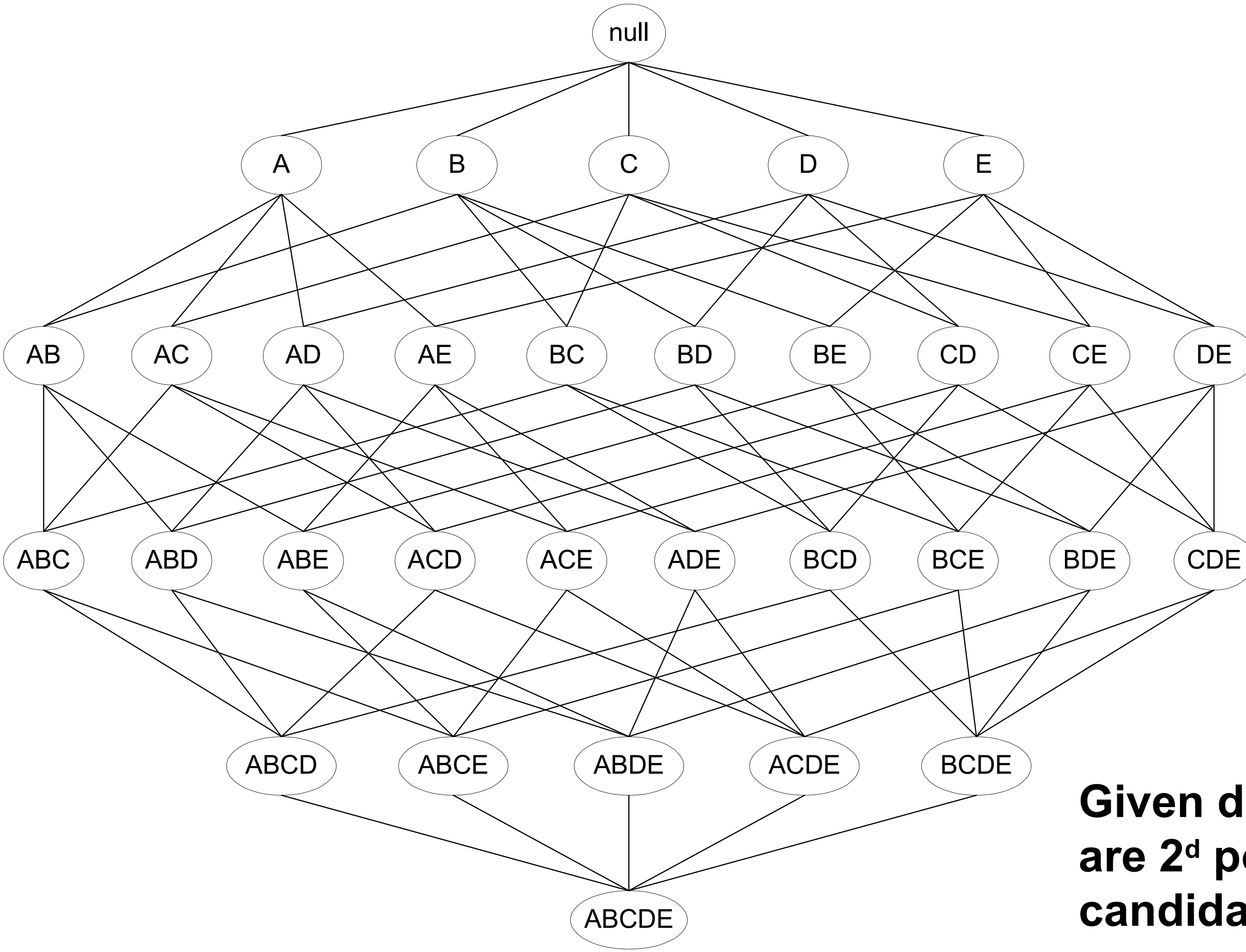
Observations:

- All the above rules are binary partitions of the same itemset:
  {Milk, Diaper, Beer}
- Rules originating from the same itemset have identical support but can have different confidence
- Thus, we may decouple the support and confidence requirements

Mining Association Rules

- Two-step approach:
  1. Frequent Itemset Generation
     – Generate all itemsets whose support $\geq$ minsup
  2. Rule Generation
     – Generate high confidence rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset
- Frequent itemset generation is still computationally expensive

Frequent Itemset Generation

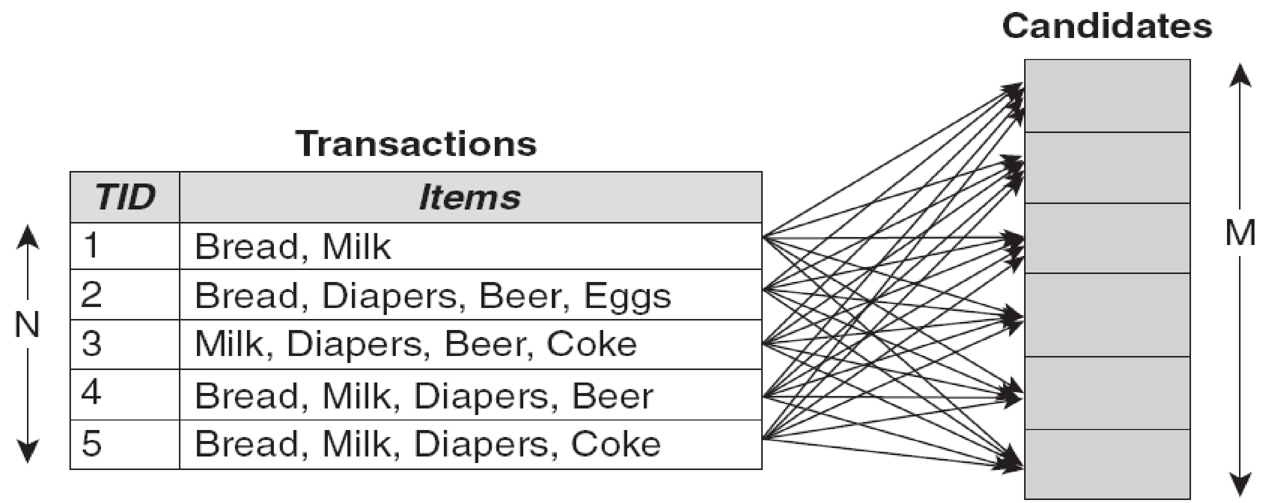


Given d items, there are 2^d possible candidate itemsets

Frequent Itemset Generation

Brute-force approach:

- Each itemset in the lattice is a candidate frequent itemset
- Count the support of each candidate by scanning the database



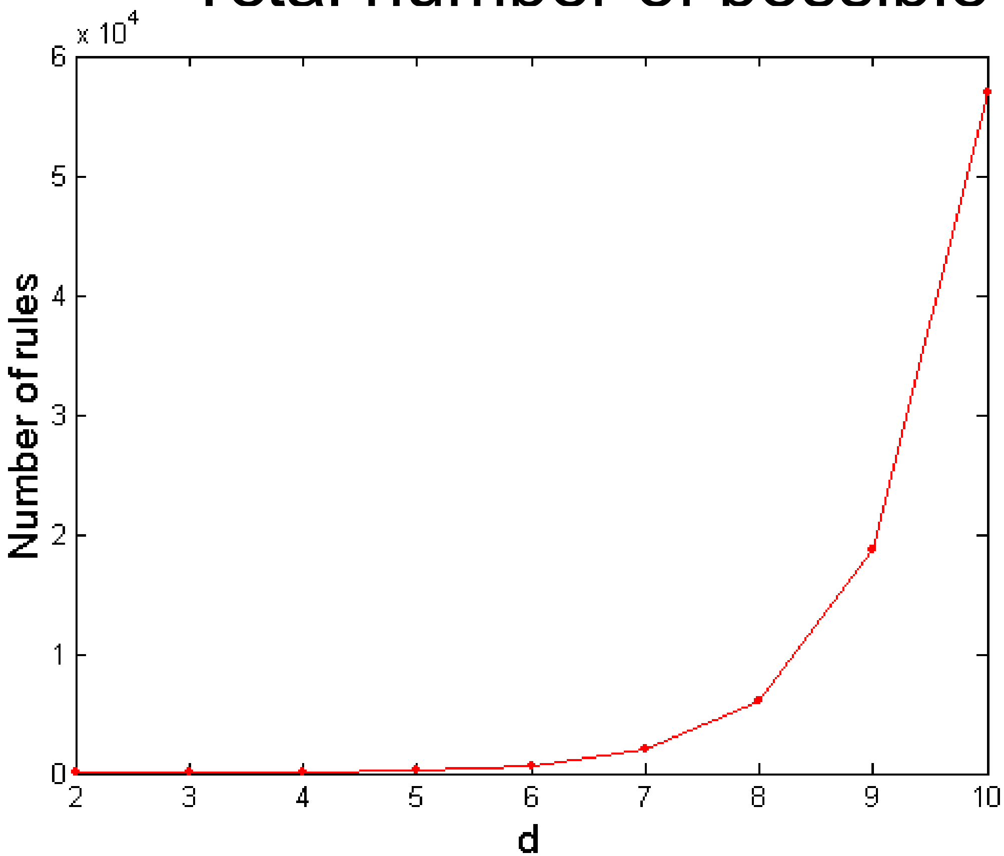
TID	Items
1	Bread, Milk
2	Bread, Diapers, Beer, Eggs
3	Milk, Diapers, Beer, Coke
4	Bread, Milk, Diapers, Beer
5	Bread, Milk, Diapers, Coke

Figure 6.2. Counting the support of candidate itemsets.

- Match each transaction against every candidate
- Complexity ~ O(NM) => Expensive since $M = 2^d$!!!

Computational Complexity

- Given d unique items:
  - Total number of itemsets = 2^d
  - Total number of possible association rules:



$$R = \sum_{k=1}^{d-1} \left[\binom{d}{k} \times \sum_{j=1}^{d-k} \binom{d-k}{j} \right] = 3^d - 2^{d+1} + 1$$

If d=6, R = 602 rules

Frequent Itemset Generation Strategies

- Reduce the number of candidates (M)
  - Complete search: $M=2^d$
  - Use pruning techniques to reduce M
- Reduce the number of transactions (N)
  - Reduce size of N as the size of itemset increases
  - Used by DHP and vertical-based mining algorithms
- Reduce the number of comparisons (NM)
  - Use efficient data structures to store the candidates or transactions
  - No need to match every candidate against every transaction

Reducing Number of Candidates

- Apriori principle:
  - If an itemset is frequent, then all of its subsets must also be frequent

- Apriori principle holds due to the following property of the support measure:

$$\forall X, Y : (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$

  - Support of an itemset never exceeds the support of its subsets
  - This is known as the anti-monotone property of support

Illustrating Apriori Principle

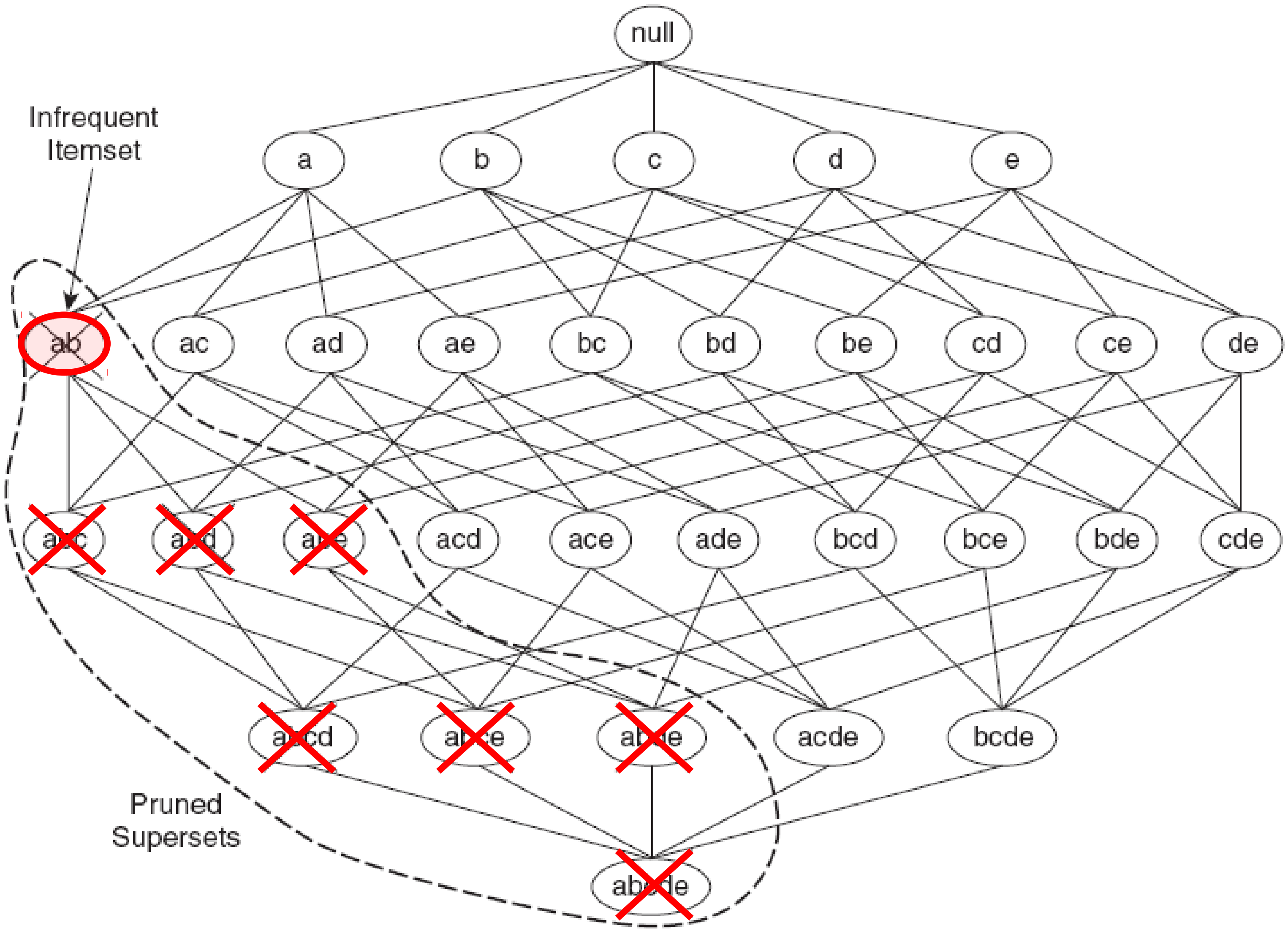


Figure 6.4. An illustration of support-based pruning. If $\{a, b\}$ is infrequent, then all supersets of $\{a, b\}$ are infrequent.

Illustrating Apriori Principle

Items (1-itemsets)

Item	Count
Bread	**4**
~~**Coke**~~	**2**
Milk	**4**
Beer	**3**
Diaper	**4**
~~**Eggs**~~	**1**

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Itemset	Count
{Bread,Milk}	**3**
~~**{Bread,Beer}**~~	**2**
{Bread,Diaper}	**3**
~~**{Milk,Beer}**~~	**2**
{Milk,Diaper}	**3**
{Beer,Diaper}	**3**

Minimum Support = 3

Triplets (3-itemsets)

Itemset	Count
{Bread,Milk,Diaper}	**3**

If every subset is considered,
$^6C_1 + {}^6C_2 + {}^6C_3 = 41$
With support-based pruning,
$6 + 6 + 1 = 13$

Apriori Algorithm

Method:

- Let k=1
- Generate frequent itemsets of length 1
- Repeat until no new frequent itemsets are identified
  - Generate length (k+1) candidate itemsets from length k frequent itemsets
  - Prune candidate itemsets containing subsets of length k that are infrequent
  - Count the support of each candidate by scanning the DB
  - Eliminate candidates that are infrequent, leaving only those that are frequent

Factors Affecting Complexity

- Choice of minimum support threshold
  - lowering support threshold results in more frequent itemsets
  - this may increase number of candidates and max length of frequent itemsets
- Dimensionality (number of items) of the data set
  - more space is needed to store support count of each item
  - if number of frequent items also increases, both computation and I/O costs may also increase
- Size of database
  - since Apriori makes multiple passes, run time of algorithm may increase with number of transactions
- Average transaction width
  - transaction width increases with denser data sets
  - This may increase max length of frequent itemsets and traversals of hash tree (number of subsets in a transaction increases with its width)

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Maximal Frequent Itemset

An itemset is maximal frequent if none of its immediate supersets is frequent

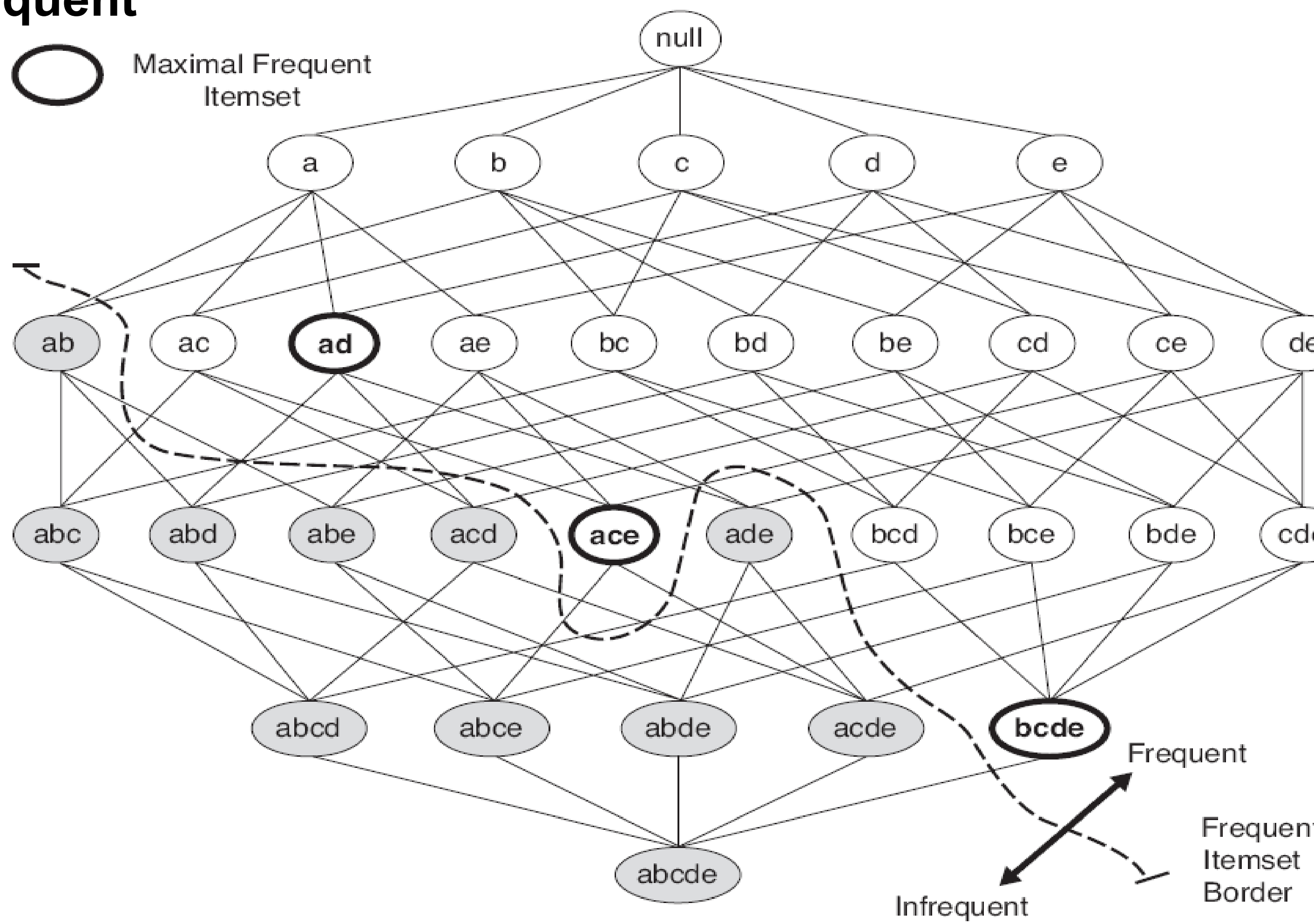


Figure 6.16. Maximal frequent itemset.

Closed Itemset

- An itemset is closed if none of its immediate supersets has the same support as the itemset (can only have smaller support -> see APRIORI principle)

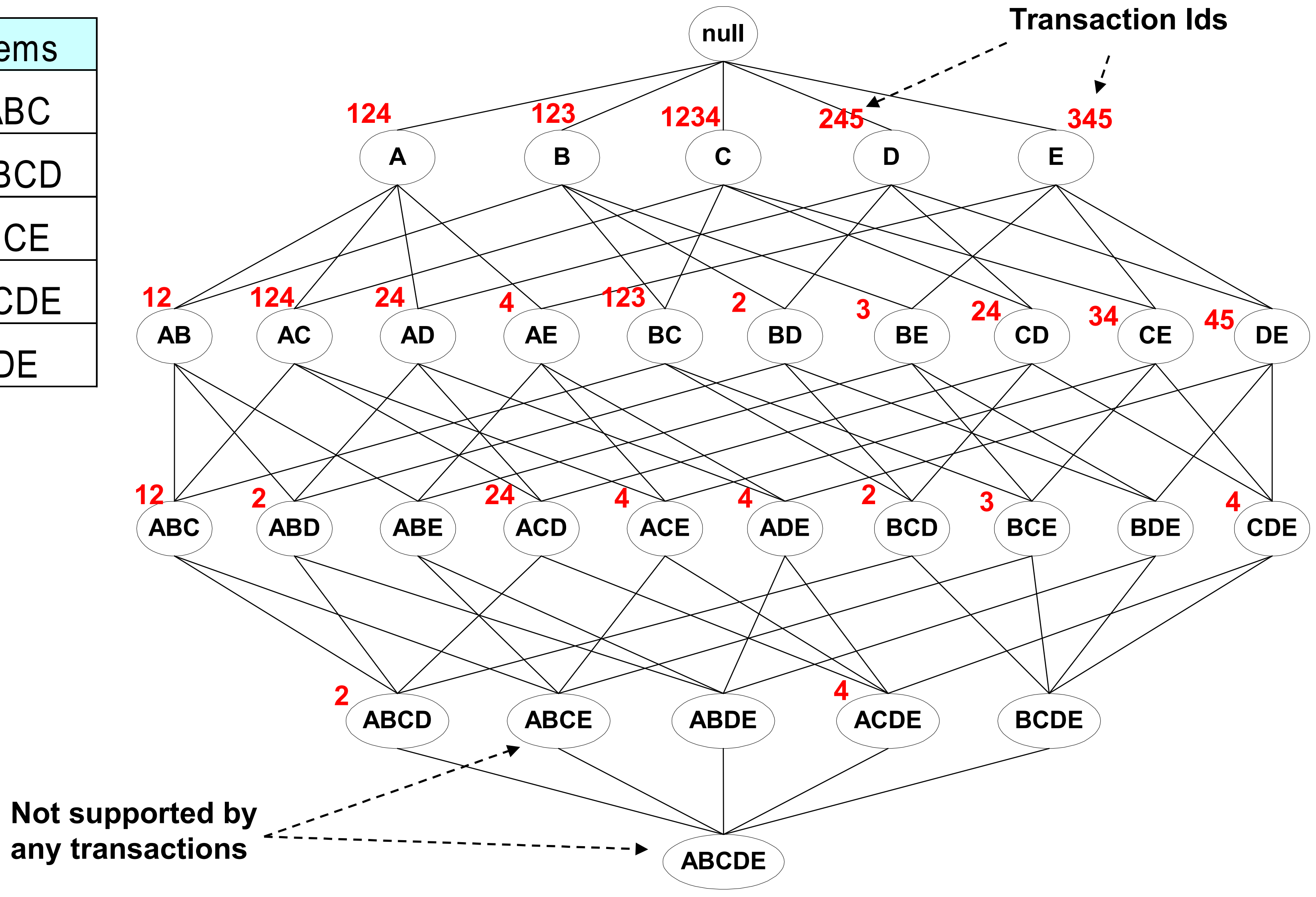
TID	Items
1	{A,B}
2	{B,C,D}
3	{A,B,C,D}
4	{A,B,D}
5	{A,B,C,D}

Itemset	Support
{A}	4
{B}	5
{C}	3
{D}	4
{A,B}	4
{A,C}	2
{A,D}	3
{B,C}	3
{B,D}	4
{C,D}	3

Itemset	Support
{A,B,C}	2
{A,B,D}	3
{A,C,D}	2
{B,C,D}	3
{A,B,C,D}	2

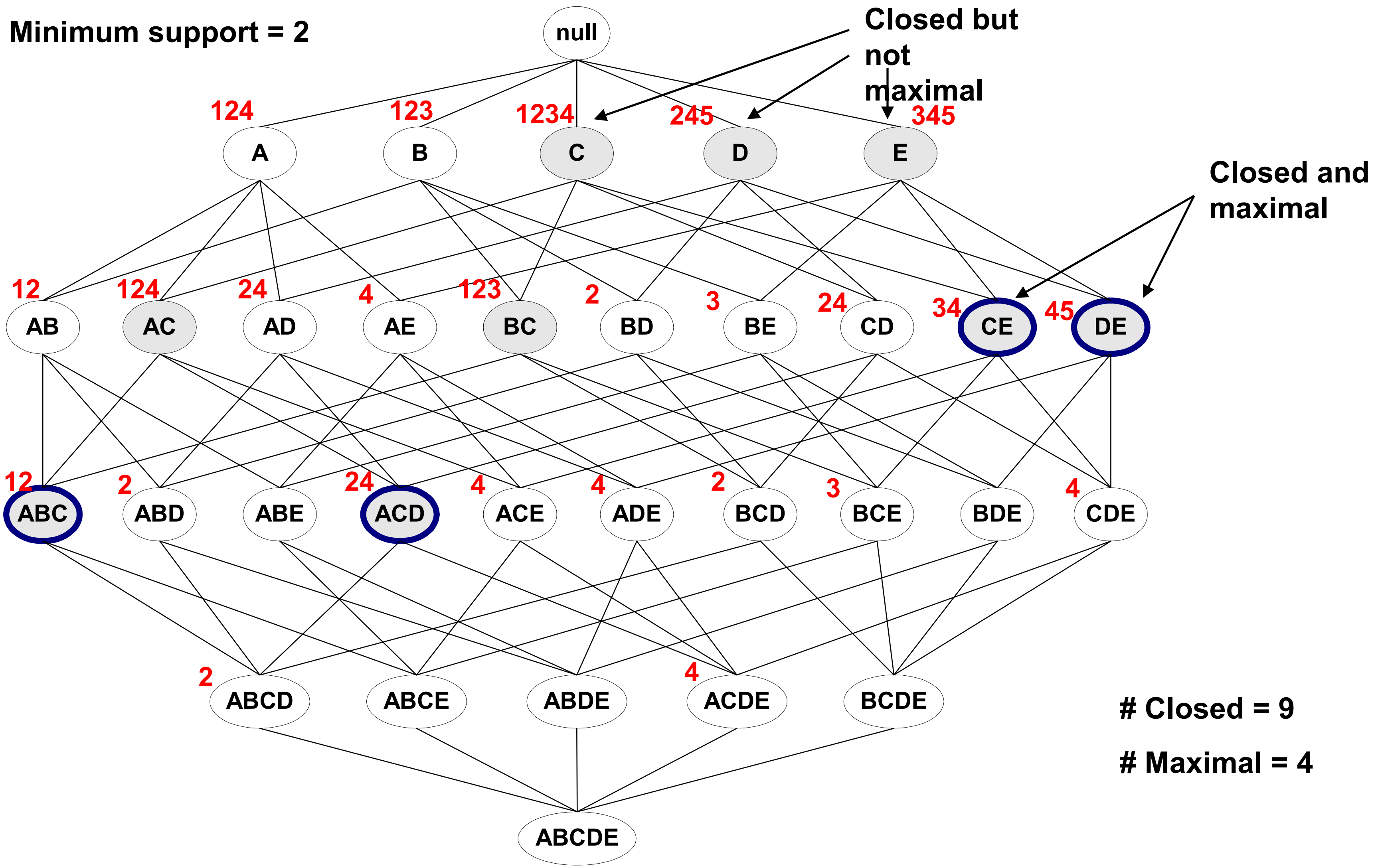
Maximal vs Closed Itemsets

TID	Items
1	ABC
2	ABCD
3	BCE
4	ACDE
5	DE



Maximal vs Closed Frequent Itemsets

Minimum support = 2



 # Closed = 9

 # Maximal = 4

Maximal vs Closed Itemsets

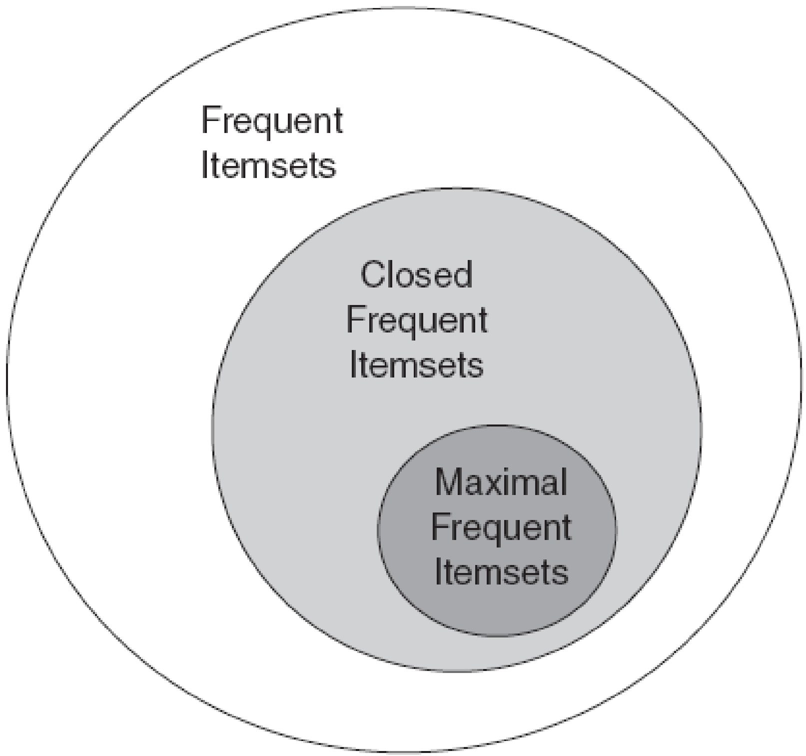


Figure 6.18. Relationships among frequent, maximal frequent, and closed frequent itemsets.

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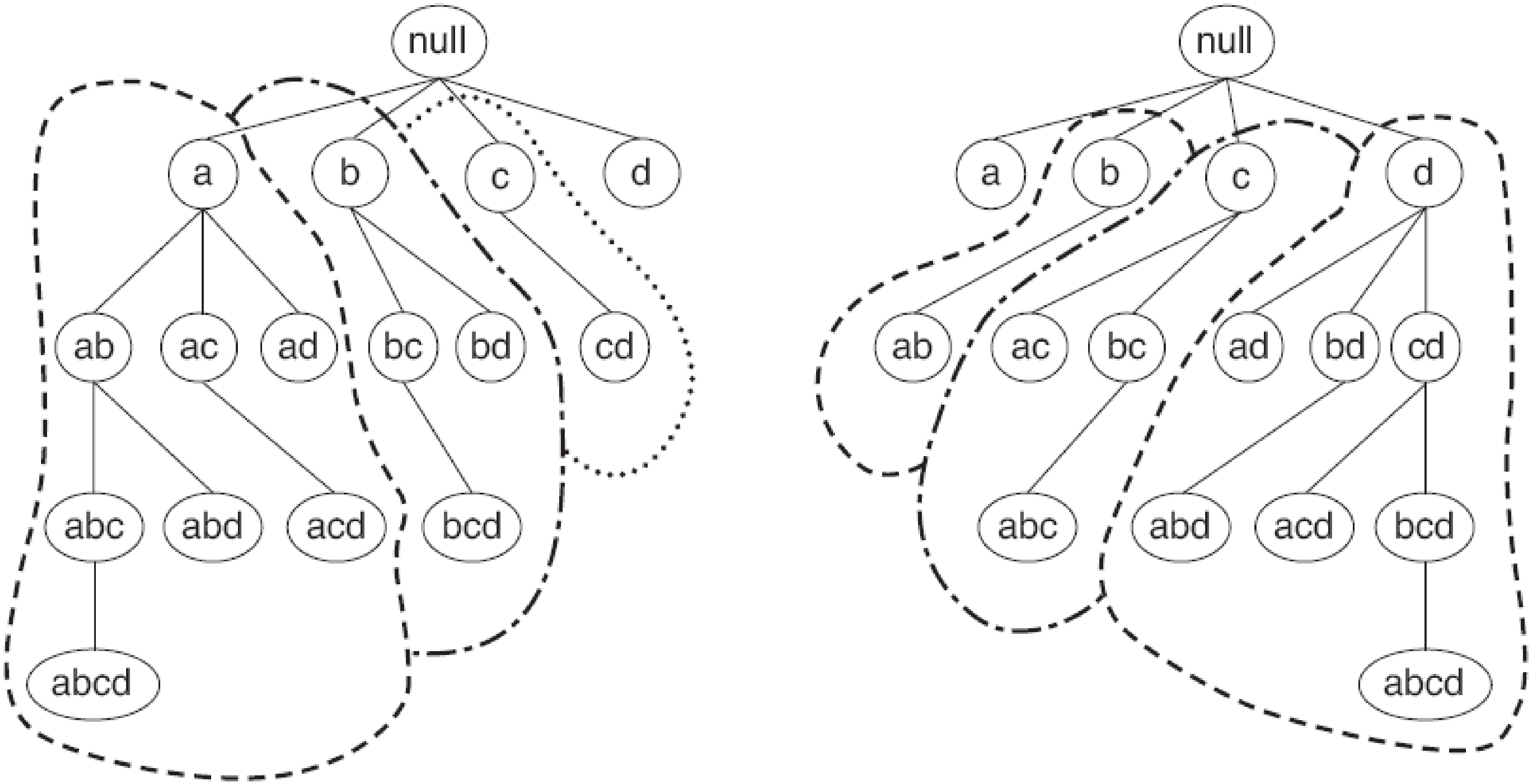
Alternative Methods for Frequent Itemset Generation

- Traversal of Itemset Lattice
  - Equivalent Classes



(a) Prefix tree.

(b) Suffix tree.

Alternative Methods for Frequent Itemset Generation

Representation of Database: horizontal vs vertical data layout

Horizontal
Data Layout

TID	Items
1	a,b,e
2	b,c,d
3	c,e
4	a,c,d
5	a,b,c,d
6	a,e
7	a,b
8	a,b,c
9	a,c,d
10	b

Vertical Data Layout

a	b	c	d	e
1	1	2	2	1
4	2	3	4	3
5	5	4	5	6
6	7	8	9	
7	8	9		
8	10			
9				

Figure 6.23. Horizontal and vertical data format.

Alternative Algorithms

- FP-growth
  - Use a compressed representation of the database using an FP-tree
  - Once an FP-tree has been constructed, it uses a recursive divide-and-conquer approach to mine the frequent itemsets
- ECLAT
  - Store transaction id-lists (vertical data layout).
  - Performs fast tid-list intersection (bit-wise XOR) to count itemset frequencies

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Rule Generation

Given a frequent itemset L, find all non-empty subsets $X=f \subset L$ and $Y=L - f$ such that $X \rightarrow Y$ satisfies the minimum confidence requirement

$$c(X \rightarrow Y) = \frac{\sigma(X \cup Y)}{\sigma(X)}$$

- If {A,B,C,D} is a frequent itemset, candidate rules:

ABC →D,	ABD →C,	ACD →B,	BCD →A,
A →BCD,	B →ACD,	C →ABD,	D →ABC
AB →CD,	AC → BD,	AD → BC,	BC →AD,
BD →AC,	CD →AB,		

If $|L| = k$, then there are $2^k - 2$ candidate association rules (ignoring $L \rightarrow \emptyset$ and $\emptyset \rightarrow L$)

Rule Generation

How to efficiently generate rules from frequent itemsets?

- In general, confidence does not have an anti-monotone property

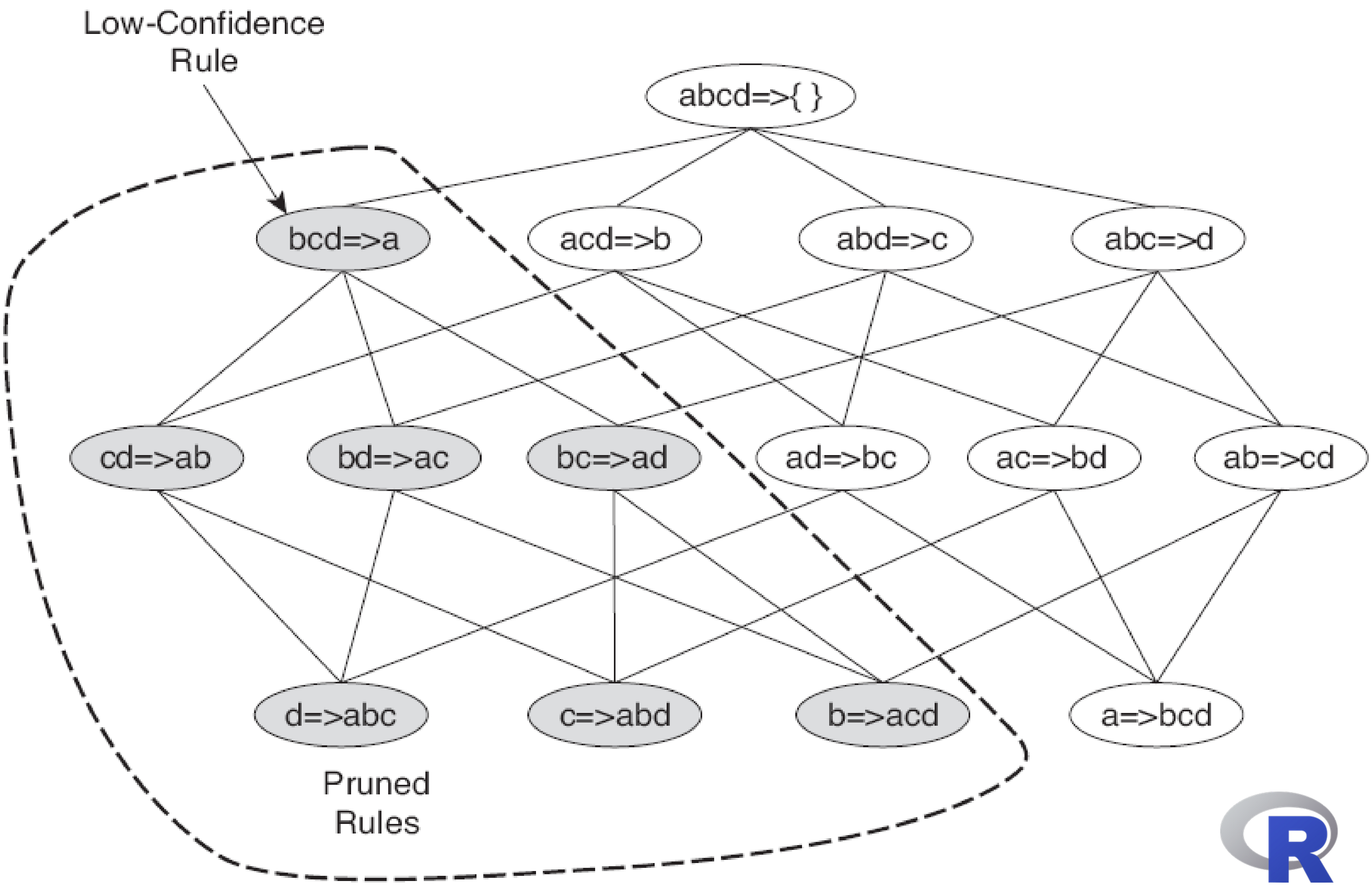
$$c(ABC \rightarrow D) \text{ can be larger or smaller than } c(AB \rightarrow D)$$

- But confidence of rules generated from the same itemset has an anti-monotone property
- e.g., $L = \{A,B,C,D\}$:

$$c(ABC \rightarrow D) \geq c(AB \rightarrow CD) \geq c(A \rightarrow BCD)$$

  - Confidence is anti-monotone w.r.t. number of items on the RHS of the rule

Rule Generation for Apriori Algorithm



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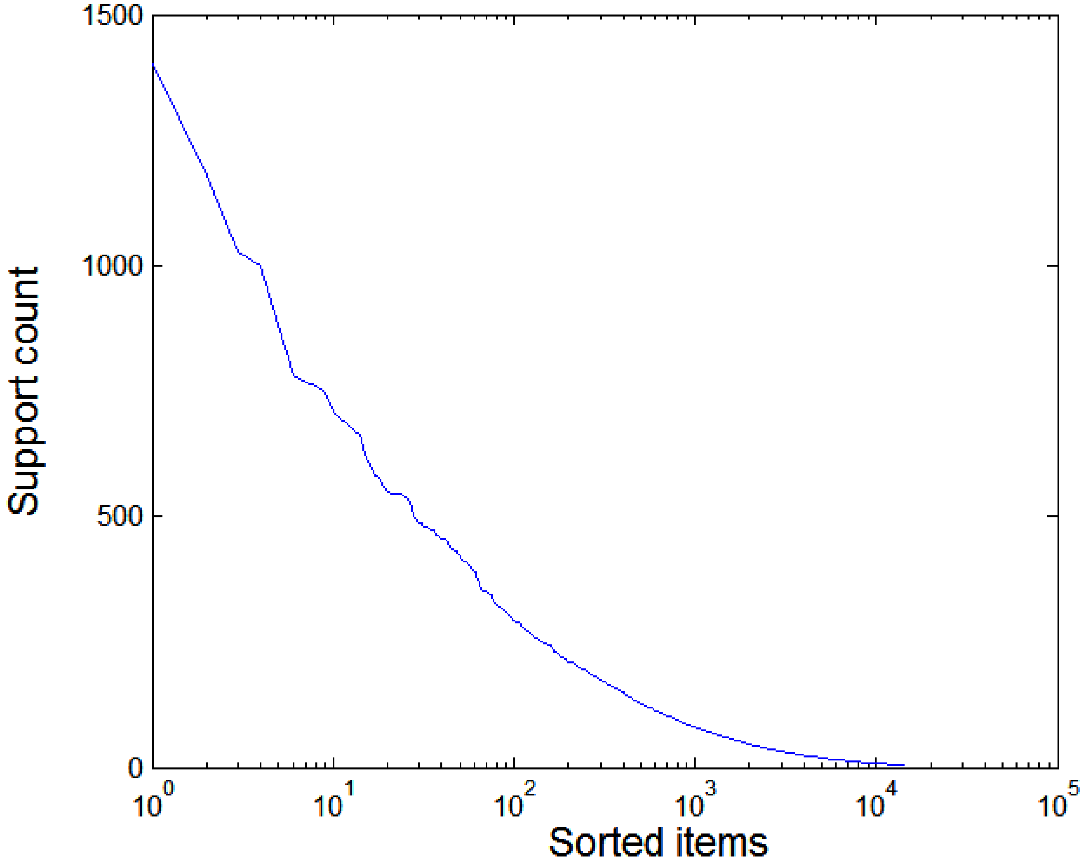
Effect of Support Distribution

- Many real data sets have skewed support distribution

Support distribution of a retail data set



Effect of Support Distribution

- How to set the appropriate *minsup* threshold?
  - If *minsup* is set too high, we could miss itemsets involving interesting rare items (e.g., expensive products)
  - If *minsup* is set too low, it is computationally expensive and the number of itemsets is very large
- Using a single minimum support threshold may not be effective

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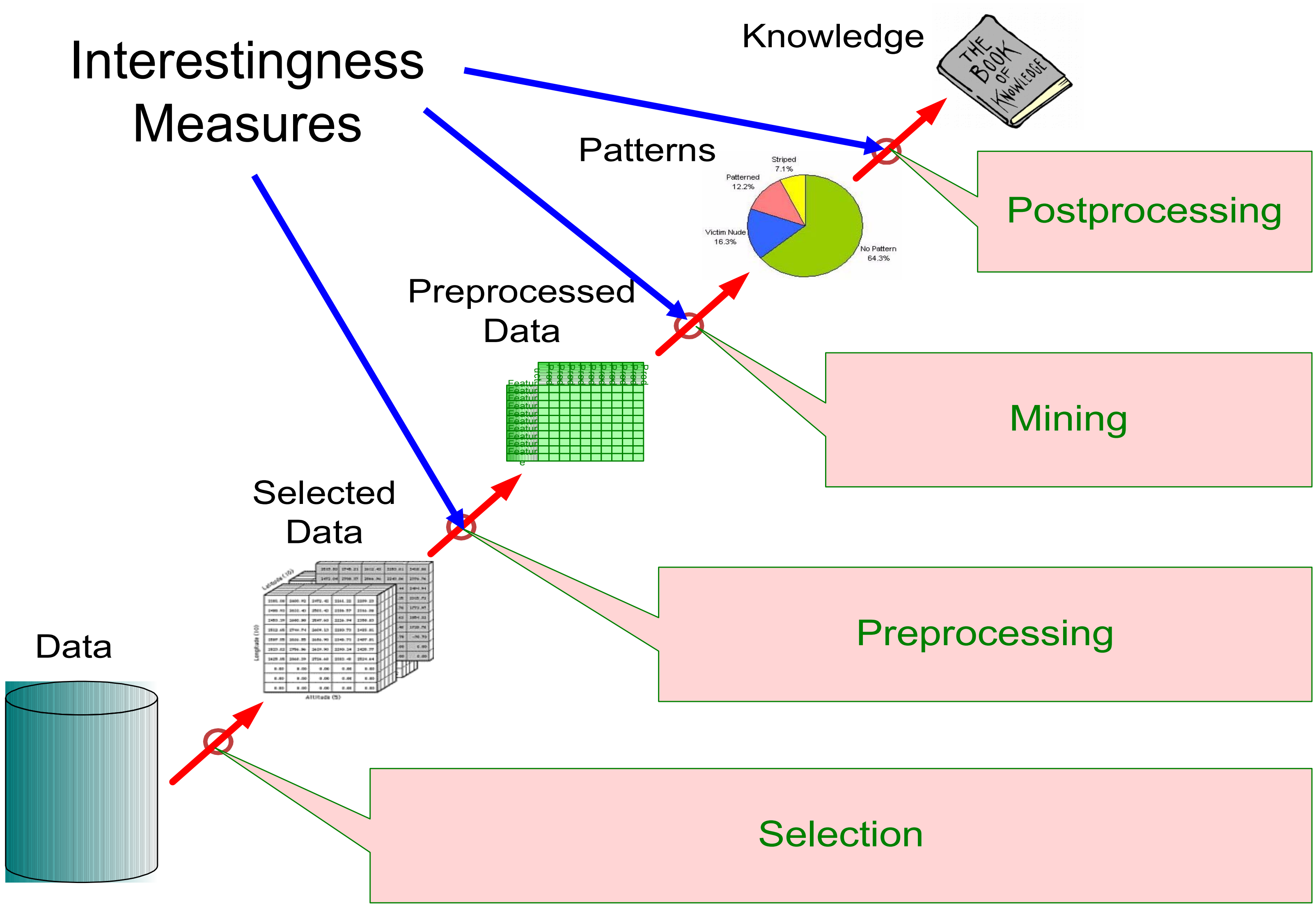
Pattern Evaluation

- Association rule algorithms tend to produce **too many rules.** Many of them are
  - uninteresting or
  - redundant

- Interestingness measures can be used to **prune/rank** the derived patterns

- A rule $\{A,B,C\} \rightarrow \{D\}$ can be considered **redundant** if $\{A,B\} \rightarrow \{D\}$ has the same or higher confidence.

Application of Interestingness Measure



Computing Interestingness Measure

Given a rule $X \rightarrow Y$, information needed to compute rule interestingness can be obtained from a contingency table

Contingency table for $X \rightarrow Y$

	Y	$\overline{Y}$	
X	f_{11}	f_{10}	f_{1+}
$\overline{X}$	f_{01}	f_{00}	f_{o+}
	f_{+1}	f_{+0}	$\|T\|$

f_{11}: support of X and Y

f_{10}: support of X and $\overline{Y}$

f_{01}: support of $\overline{X}$ and $\overline{Y}$

f_{00}: support of $\overline{X}$ and Y

error

Used to define various measures

e.g., support, confidence, lift, Gini, J-measure, etc.

$\text{sup}(\{X, Y\}) = f_{11} / |T|$ estimates $P(X, Y)$

$\text{conf}(X\text{->}Y) = f_{11} / f_{1+}$ estimates $P(Y \mid X)$

Drawback of Confidence

	Coffee	$\overline{\text{Coffee}}$	
Tea	15	5	20
$\overline{\text{Tea}}$	75	5	80
	90	10	100

Association Rule: Tea → Coffee

Support = P(Coffee, Tea) = 15/100 = 0.15

Confidence= P(Coffee | Tea) = 15/20 = 0.75

but P(Coffee) = 90/100 = 0.9

⇒ Although confidence is high, rule is misleading

⇒ P(Coffee|$\overline{\text{Tea}}$) = 75/80 = **0.9375**

Statistical Independence

Population of 1000 students

- 600 students know how to swim (S)
- 700 students know how to bike (B)
- 450 students know how to swim and bike (S,B)

- P(S,B) = 450/1000 = 0.45 (observed joint prob.)
- P(S) $\times$ P(B) = 0.6 $\times$ 0.7 = 0.42 (expected under indep.)

- P(S,B) = P(S) $\times$ P(B) => Statistical independence
- P(S,B) > P(S) $\times$ P(B) => Positively correlated
- P(S,B) < P(S) $\times$ P(B) => Negatively correlated

Statistical-based Measures

Measures that take statistical dependence into account for rule: $X \rightarrow Y$

$$\left.\begin{aligned} Lift = Interest &= \frac{P(Y|X)}{P(Y)} = \frac{P(X,Y)}{P(X)P(Y)} \\ PS &= P(X,Y) - P(X)P(Y) \end{aligned}\right\} \text{Deviation from independence}$$

$$\Phi - coefficient = \frac{P(X,Y) - P(X)P(Y)}{\sqrt{P(X)[1-P(X)]P(Y)[1-P(Y)]}}$$

Correlation

Example: Lift/Interest

	Coffee	$\overline{\text{Coffee}}$	
Tea	15	5	20
$\overline{\text{Tea}}$	75	5	80
	90	10	100

Association Rule: Tea → Coffee

Conf(Tea → Coffee)= P(Coffee|Tea) = P(Coffee,Tea)/P(Tea)
= .15/.2 = 0.75

but P(Coffee) = 0.9

⇒ Lift(Tea → Coffee) = P(Coffee,Tee)/(P(Coffee)P(Tee))
= .15/(.9 x .2) = **0.8333**

Note: Lift < 1, therefore Coffee and Tea are negatively associated

There are lots of measures proposed in the literature

Some measures are good for certain applications, but not for others

What criteria should we use to determine whether a measure is good or bad?

What about Apriori-style support based pruning? How does it affect these measures?

#	Measure	Formula
1	ϕ-coefficient	$\frac{P(A,B)-P(A)P(B)}{\sqrt{P(A)P(B)(1-P(A))(1-P(B))}}$
2	Goodman-Kruskal's (λ)	$\frac{\sum_j \max_k P(A_j,B_k)+\sum_k \max_j P(A_j,B_k)-\max_j P(A_j)-\max_k P(B_k)}{2-\max_j P(A_j)-\max_k P(B_k)}$
3	Odds ratio (α)	$\frac{P(A,B)P(\overline{A},\overline{B})}{P(A,\overline{B})P(\overline{A},B)}$
4	Yule's Q	$\frac{P(A,B)P(\overline{AB})-P(A,\overline{B})P(\overline{A},B)}{P(A,B)P(\overline{AB})+P(A,\overline{B})P(\overline{A},B)} = \frac{\alpha-1}{\alpha+1}$
5	Yule's Y	$\frac{\sqrt{P(A,B)P(\overline{AB})}-\sqrt{P(A,\overline{B})P(\overline{A},B)}}{\sqrt{P(A,B)P(\overline{AB})}+\sqrt{P(A,\overline{B})P(\overline{A},B)}} = \frac{\sqrt{\alpha}-1}{\sqrt{\alpha}+1}$
6	Kappa (κ)	$\frac{P(A,B)+P(\overline{A},\overline{B})-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A)P(B)-P(\overline{A})P(\overline{B})}$
7	Mutual Information (M)	$\frac{\sum_i \sum_j P(A_i,B_j)\log\frac{P(A_i,B_j)}{P(A_i)P(B_j)}}{\min(-\sum_i P(A_i)\log P(A_i),-\sum_j P(B_j)\log P(B_j))}$
8	J-Measure (J)	$\max\Big(P(A,B)\log(\frac{P(B\mid A)}{P(B)})+P(A\overline{B})\log(\frac{P(\overline{B}\mid A)}{P(\overline{B})}),$ $P(A,B)\log(\frac{P(A\mid B)}{P(A)})+P(\overline{A}B)\log(\frac{P(\overline{A}\mid B)}{P(A)})\Big)$
9	Gini index (G)	$\max\Big(P(A)[P(B\mid A)^2+P(\overline{B}\mid A)^2]+P(\overline{A})[P(B\mid\overline{A})^2+P(\overline{B}\mid\overline{A})^2]$ $-P(B)^2-P(\overline{B})^2,$ $P(B)[P(A\mid B)^2+P(\overline{A}\mid B)^2]+P(\overline{B})[P(A\mid\overline{B})^2+P(\overline{A}\mid\overline{B})^2]$ $-P(A)^2-P(\overline{A})^2\Big)$
10	Support (s)	$P(A,B)$
11	Confidence (c)	$\max(P(B\mid A),P(A\mid B))$
12	Laplace (L)	$\max\Big(\frac{NP(A,B)+1}{NP(A)+2},\frac{NP(A,B)+1}{NP(B)+2}\Big)$
13	Conviction (V)	$\max\Big(\frac{P(A)P(\overline{B})}{P(A\overline{B})},\frac{P(B)P(\overline{A})}{P(B\overline{A})}\Big)$
14	Interest (I)	$\frac{P(A,B)}{P(A)P(B)}$
15	cosine (IS)	$\frac{P(A,B)}{\sqrt{P(A)P(B)}}$
16	Piatetsky-Shapiro's (PS)	$P(A,B)-P(A)P(B)$
17	Certainty factor (F)	$\max\Big(\frac{P(B\mid A)-P(B)}{1-P(B)},\frac{P(A\mid B)-P(A)}{1-P(A)}\Big)$
18	Added Value (AV)	$\max(P(B\mid A)-P(B),P(A\mid B)-P(A))$
19	Collective strength (S)	$\frac{P(A,B)+P(\overline{AB})}{P(A)P(B)+P(\overline{A})P(\overline{B})}\times\frac{1-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A,B)-P(\overline{AB})}$
20	Jaccard (ζ)	$\frac{P(A,B)}{P(A)+P(B)-P(A,B)}$
21	Klosgen (K)	$\sqrt{P(A,B)}\max(P(B\mid A)-P(B),P(A\mid B)-P(A))$

Comparing Different Measures

10 examples of contingency tables:

Example	f_{11}	f_{10}	f_{01}	f_{00}
E1	8123	83	424	1370
E2	8330	2	622	1046
E3	9481	94	127	298
E4	3954	3080	5	2961
E5	2886	1363	1320	4431
E6	1500	2000	500	6000
E7	4000	2000	1000	3000
E8	4000	2000	2000	2000
E9	1720	7121	5	1154
E10	61	2483	4	7452

Rankings of contingency tables using various measures:

support & confidence

#	ϕ	λ	α	Q	Y	κ	M	J	G	s	c	L	V	I	IS	PS	F	AV	S	ζ	K
E1	1	1	3	3	3	1	2	2	1	3	5	5	4	6	2	2	4	6	1	2	5
E2	2	2	1	1	1	2	1	3	2	2	1	1	1	8	3	5	1	8	2	3	6
E3	3	3	4	4	4	3	3	8	7	1	4	4	6	10	1	8	6	10	3	1	10
E4	4	7	2	2	2	5	4	1	3	6	2	2	2	4	4	1	2	3	4	5	1
E5	5	4	8	8	8	4	7	5	4	7	9	9	9	3	6	3	9	4	5	6	3
E6	6	6	7	7	7	7	6	4	6	9	8	8	7	2	8	6	7	2	7	8	2
E7	7	5	9	9	9	6	8	6	5	4	7	7	8	5	5	4	8	5	6	4	4
E8	8	9	10	10	10	8	10	10	8	4	10	10	10	9	7	7	10	9	8	7	9
E9	9	9	5	5	5	9	9	7	9	8	3	3	3	7	9	9	3	7	9	9	8
E10	10	8	6	6	6	10	5	9	10	10	6	6	5	1	10	10	5	1	10	[illegible]	[illegible]

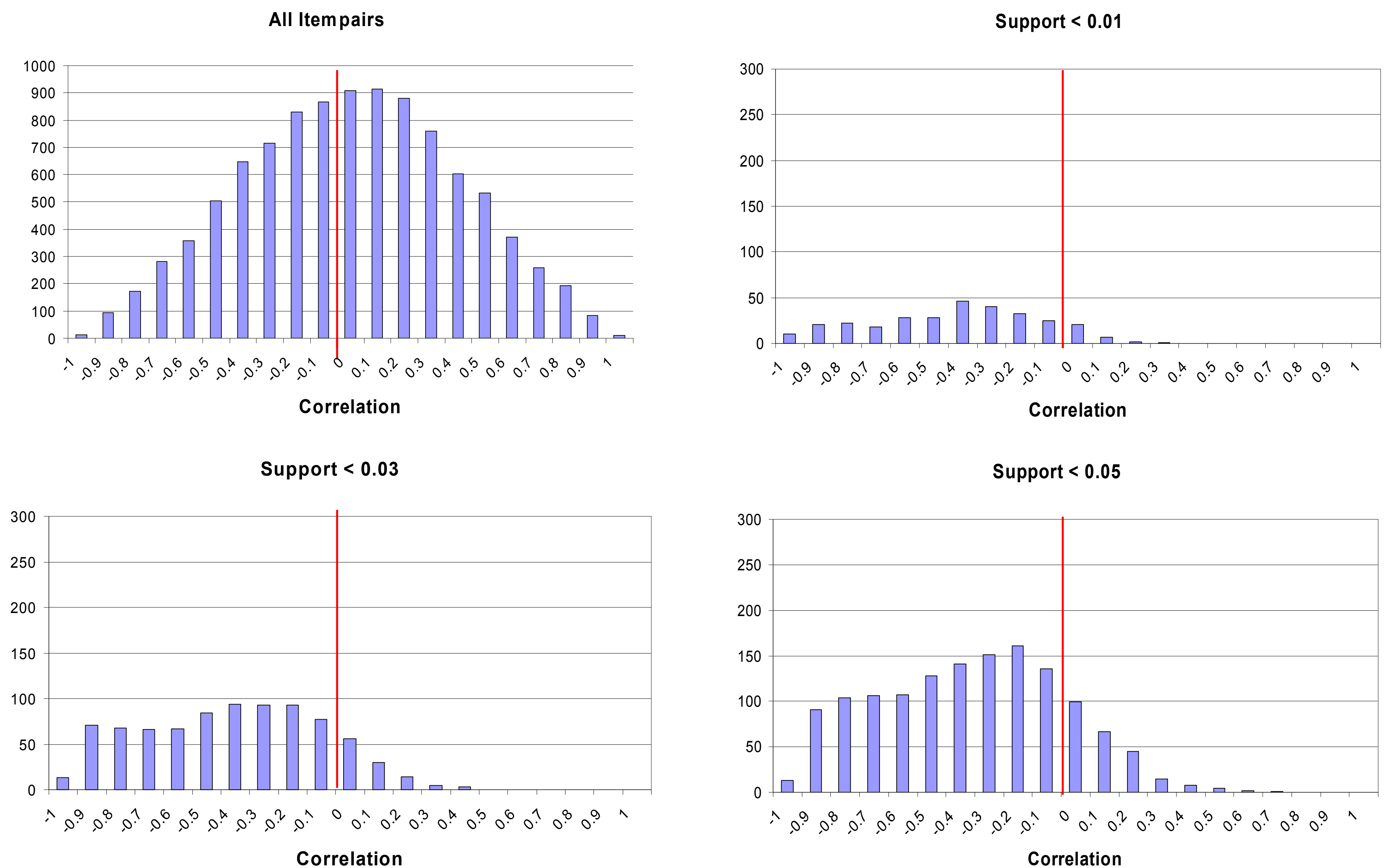
lift

Support-based Pruning

- Most of the association rule mining algorithms use support measure to prune rules and itemsets

- Study effect of support pruning on correlation of itemsets
  - Generate 10,000 random contingency tables
  - Compute support and pairwise correlation for each table
  - Apply support-based pruning and examine the tables that are removed

Effect of Support-based Pruning



Support-based pruning eliminates mostly negatively correlated itemsets

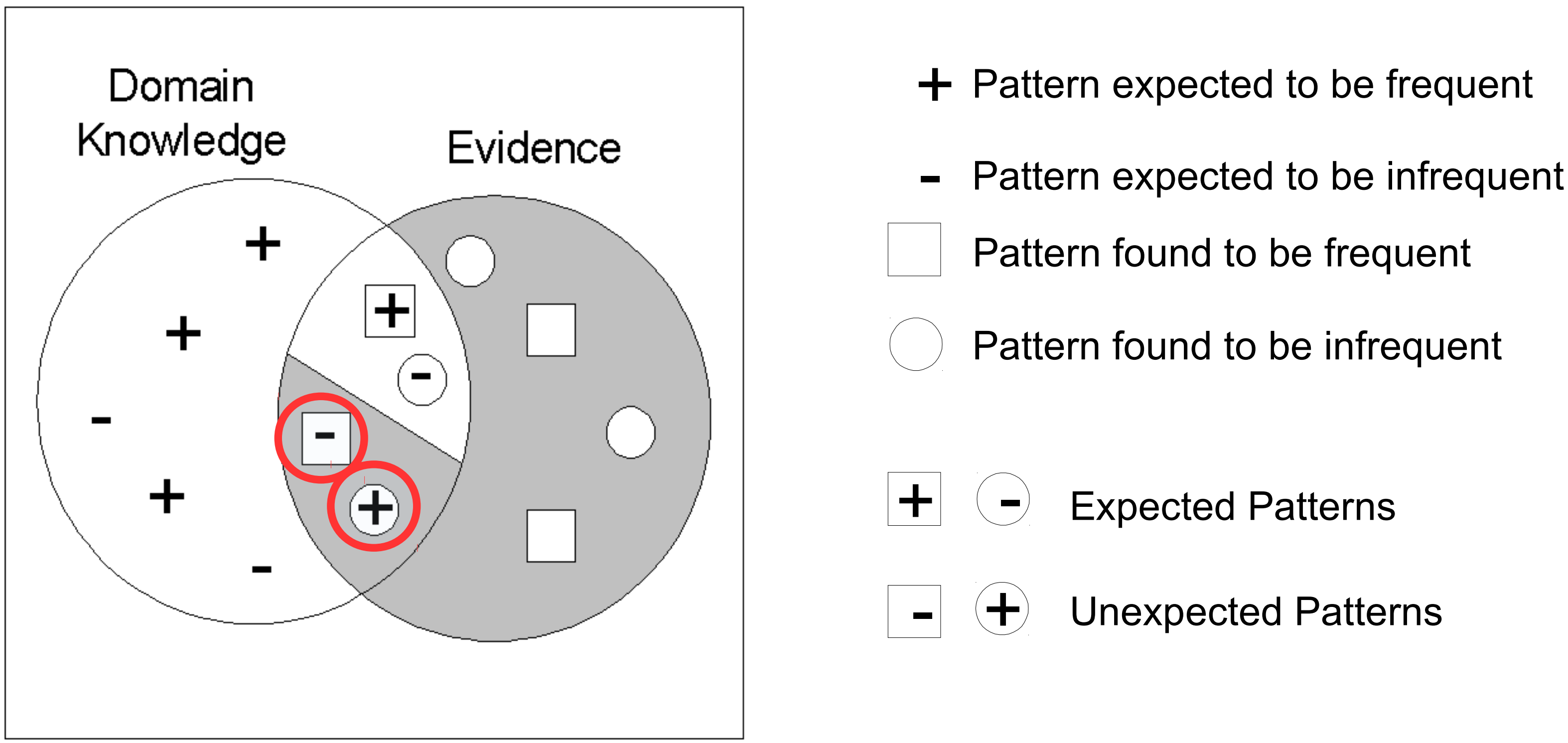
Subjective Interestingness Measure

- Objective measure:
  - Rank patterns based on statistics computed from data
  - e.g., 21 measures of association (support, confidence, Laplace, Gini, mutual information, Jaccard, etc).

- Subjective measure:
  - Rank patterns according to user's interpretation
    - A pattern is subjectively interesting if it **contradicts the expectation** of a user (Silberschatz & Tuzhilin)
    - A pattern is subjectively interesting if it is **actionable** (Silberschatz & Tuzhilin)

Interestingness via Unexpectedness

- Need to model expectation of users (domain knowledge)



- Need to combine expectation of users with evidence from data (i.e., extracted patterns)

Applications for Association Rules

- **Market Basket Analysis**
  Marketing & Retail. E.g., frequent itemsets give information about "other customer who bought this item also bought X"

- **Exploratory Data Analysis**
  Find correlation in very large (= many transactions), high-dimensional (= many items) data

- **Intrusion Detection**
  Rules with low support but very high lift

- **Build Rule-based Classifiers**
  Class association rules (CARs)