CSE 7/5337: Information Retrieval and Web Search Scoring, term weighting, the vector space model (IIR 6)

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These slides are largely based on the slides by Hinrich Schütze Institute for Natural Language Processing, University of Stuttgart http://informationretrieval.org

Spring 2012

Overview

- Recap
- Why ranked retrieval?
- Term frequency
- 4 tf-idf weighting
- 5 The vector space model

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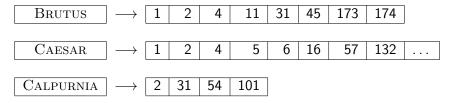
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Inverted index

For each term t, we store a list of all documents that contain t.



:

dictionary

postings

Intersecting two postings lists

BRUTUS
$$\longrightarrow$$
 1 \longrightarrow 2 \longrightarrow 4 \longrightarrow 11 \longrightarrow 31 \longrightarrow 45 \longrightarrow 173 \longrightarrow 174

CALPURNIA \longrightarrow 2 \longrightarrow 31 \longrightarrow 54 \longrightarrow 101

Intersection \Longrightarrow 2 \longrightarrow 31

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Constructing the inverted index: Sort postings

term	docID		term	docID
1	1		ambitio	us 2
did	1		2	
enact	1		brutus	1
julius	1		brutus	2
caesar	1		capitol	1
I	1		caesar	1
was	1		caesar	2
killed	1		caesar	2
i'	1		did	1
the	1		enact	1
capitol	1		hath	1
brutus	1		1	1
killed	1		1	1
me	1	\Longrightarrow	i'	1
so	2	$\overline{}$	it	2
let	2		julius	1
it	2		killed	1
be	2		killed	1
with	2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2		let	2
caesar	2		me	1
the	2		noble	2
noble	2		so	2
brutus	2		the	1
hath	2		the	2
told	2		told	2
you	2		you	2 1
caesar	2		was	1
was	2		was	2
ambitio	us 2		with	2

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Westlaw: Example queries

Information need: Information on the legal theories involved in preventing the disclosure of trade secrets by employees formerly employed by a competing company

Query: "trade secret" /s disclos! /s prevent /s employe!

Information need: Requirements for disabled people to be able to access a workplace

Query: disab! / p access! / s work-site work-place (employment / 3 place)

Information need: Cases about a host's responsibility for drunk guests Query: host! /p (responsib! liab!) /p (intoxicat! drunk!) /p guest

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Does Google use the Boolean model?

- On Google, the default interpretation of a query $[w_1 \ w_2 \ \dots \ w_n]$ is w_1 AND w_2 AND ... AND w_n
- Cases where you get hits that do not contain one of the w_i:
 - anchor text
 - \triangleright page contains variant of w_i (morphology, spelling correction, synonym)
 - long queries (n large)
 - boolean expression generates very few hits
- Simple Boolean vs. Ranking of result set
 - Simple Boolean retrieval returns matching documents in no particular order.
 - Google (and most well designed Boolean engines) rank the result set they rank good hits (according to some estimator of relevance) higher than bad hits.

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Type/token distinction

- Token an instance of a word or term occurring in a document
- Type an equivalence class of tokens
- In June, the dog likes to chase the cat in the barn.
- 12 word tokens, 9 word types

Problems in tokenization

- What are the delimiters? Space? Apostrophe? Hyphen?
- For each of these: sometimes they delimit, sometimes they don't.
- No whitespace in many languages! (e.g., Chinese)
- No whitespace in Dutch, German, Swedish compounds (Lebensversicherungsgesellschaftsangestellter)

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Problems with equivalence classing

- A term is an equivalence class of tokens.
- How do we define equivalence classes?
- Numbers (3/20/91 vs. 20/3/91)
- Case folding
- Stemming, Porter stemmer
- Morphological analysis: inflectional vs. derivational
- Equivalence classing problems in other languages
 - More complex morphology than in English
 - ► Finnish: a single verb may have 12,000 different forms
 - Accents, umlauts

Positional indexes

- Postings lists in a nonpositional index: each posting is just a docID
- Postings lists in a positional index: each posting is a docID and a list of positions
- Example query: "to1 be2 or3 not4 to5 be6"

```
TO, 993427:

$\langle 1: \langle 7, 18, 33, 72, 86, 231 \rangle;
2: \langle 1, 17, 74, 222, 255 \rangle;
4: \langle 8, 16, 190, 429, 433 \rangle;
5: \langle 363, 367 \rangle;
7: \langle 13, 23, 191 \rangle; \ldots \rangle

BE, 178239:
$\langle 1: \langle 17, 25 \rangle;
4: \langle 17, 191, 291, 430, 434 \rangle;
5: \langle 14, 19, 101 \rangle; \ldots \rangle
$\langle$
```

Document 4 is a match!

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Positional indexes

- With a positional index, we can answer
 - phrase queries
 - proximity queries

Take-away today

- Ranking search results: why it is important (as opposed to just presenting a set of unordered Boolean results)
- Term frequency: This is a key ingredient for ranking.
- Tf-idf ranking: best known traditional ranking scheme
- Vector space model: One of the most important formal models for information retrieval (along with Boolean and probabilistic models)

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Ranked retrieval

- Thus far, our queries have been Boolean.
 - Documents either match or don't.
- Good for expert users with precise understanding of their needs and of the collection.
- Also good for applications: Applications can easily consume 1000s of results.
- Not good for the majority of users
- Most users are not capable of writing Boolean queries . . .
 - ...or they are, but they think it's too much work.
- Most users don't want to wade through 1000s of results.
- This is particularly true of web search.

Problem with Boolean search: Feast or famine

- Boolean queries often result in either too few (=0) or too many (1000s) results.
- Query 1 (boolean conjunction): [standard user dlink 650]
 - ightharpoonup ightharpoonup 200,000 hits feast
- Query 2 (boolean conjunction): [standard user dlink 650 no card found]
 - \rightarrow 0 hits famine
- In Boolean retrieval, it takes a lot of skill to come up with a query that produces a manageable number of hits.

Feast or famine: No problem in ranked retrieval

- With ranking, large result sets are not an issue.
- Just show the top 10 results
- Doesn't overwhelm the user
- Premise: the ranking algorithm works: More relevant results are ranked higher than less relevant results.

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Scoring as the basis of ranked retrieval

- We wish to rank documents that are more relevant higher than documents that are less relevant.
- How can we accomplish such a ranking of the documents in the collection with respect to a query?
- Assign a score to each query-document pair, say in [0, 1].
- This score measures how well document and query "match".

Query-document matching scores

- How do we compute the score of a query-document pair?
- Let's start with a one-term query.
- If the query term does not occur in the document: score should be 0.
- The more frequent the query term in the document, the higher the score
- We will look at a number of alternatives for doing this.

Take 1: Jaccard coefficient

- A commonly used measure of overlap of two sets
- Let A and B be two sets
- Jaccard coefficient:

$$JACCARD(A, B) = \frac{|A \cap B|}{|A \cup B|}$$

$$(A \neq \emptyset \text{ or } B \neq \emptyset)$$

- JACCARD(A, A) = 1
- JACCARD(A, B) = 0 if $A \cap B = 0$
- A and B don't have to be the same size.
- Always assigns a number between 0 and 1.

Jaccard coefficient: Example

- What is the query-document match score that the Jaccard coefficient computes for:
 - Query: "ides of March"
 - Document "Caesar died in March"
 - ▶ JACCARD(q, d) = 1/6

What's wrong with Jaccard?

- It doesn't consider term frequency (how many occurrences a term has).
- Rare terms are more informative than frequent terms. Jaccard does not consider this information.
- We need a more sophisticated way of normalizing for the length of a document.
- Later in this lecture, we'll use $|A \cap B|/\sqrt{|A \cup B|}$ (cosine) . . .
- ullet . . . instead of $|A\cap B|/|A\cup B|$ (Jaccard) for length normalization.

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Binary incidence matrix

	Anthony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth	
Anthony	1	1	0	0	0	1	
Brutus	1	1	0	1	0	0	
Caesar	1	1	0	1	1	1	
Calpurnia	0	1	0	0	0	0	
CLEOPATRA	1	0	0	0	0	0	
MERCY	1	0	1	1	1	1	
WORSER	1	0	1	1	1	0	

Each document is represented as a binary vector $\in \{0,1\}^{|V|}$.

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Count matrix

	Anthony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth	
Anthony	157	73	0	0	0	1	
Brutus	4	157	0	2	0	0	
Caesar	232	227	0	2	1	5	
Calpurnia	0	10	0	0	0	0	
CLEOPATRA	57	0	0	0	0	0	
MERCY	2	0	3	8	5	8	
WORSER	2	0	1	1	1	0	

. . .

Each document is now represented as a count vector $\in \mathbb{N}^{|V|}$.

Bag of words model

- We do not consider the order of words in a document.
- John is quicker than Mary and Mary is quicker than John are represented the same way.
- This is called a bag of words model.
- In a sense, this is a step back: The positional index was able to distinguish these two documents.
- We will look at "recovering" positional information later in this course.
- For now: bag of words model

Term frequency tf

- The term frequency $tf_{t,d}$ of term t in document d is defined as the number of times that t occurs in d.
- We want to use tf when computing query-document match scores.
- But how?
- Raw term frequency is not what we want because:
- A document with tf = 10 occurrences of the term is more relevant than a document with tf = 1 occurrence of the term.
- But not 10 times more relevant.
- Relevance does not increase proportionally with term frequency.

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Instead of raw frequency: Log frequency weighting

• The log frequency weight of term t in d is defined as follows

$$\mathbf{w}_{t,d} = \left\{ \begin{array}{ll} 1 + \log_{10} \mathsf{tf}_{t,d} & \mathsf{if} \ \mathsf{tf}_{t,d} > 0 \\ 0 & \mathsf{otherwise} \end{array} \right.$$

- $\mathsf{tf}_{t,d} \to \mathsf{w}_{t,d}$: $0 \rightarrow 0$, $1 \rightarrow 1$, $2 \rightarrow 1.3$, $10 \rightarrow 2$, $1000 \rightarrow 4$, etc.
- Score for a document-query pair: sum over terms t in both q and d: tf -matching-score $(q, d) = \sum_{t \in q \cap d} (1 + \log \mathsf{tf}_{t, d})$
- The score is 0 if none of the query terms is present in the document.

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Exercise

- Compute the Jaccard matching score and the tf matching score for the following query-document pairs.
- q: [information on cars] d: "all you've ever wanted to know about cars"
- q: [information on cars] d: "information on trucks, information on planes, information on trains"
- q: [red cars and red trucks] d: "cops stop red cars more often"

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Frequency in document vs. frequency in collection

- In addition, to term frequency (the frequency of the term in the document) . . .
- ... we also want to use the frequency of the term in the collection for weighting and ranking.

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Desired weight for rare terms

- Rare terms are more informative than frequent terms.
- Consider a term in the query that is rare in the collection (e.g., ARACHNOCENTRIC).
- A document containing this term is very likely to be relevant.
- ullet \to We want high weights for rare terms like ARACHNOCENTRIC.

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Desired weight for frequent terms

- Frequent terms are less informative than rare terms.
- Consider a term in the query that is frequent in the collection (e.g., GOOD, INCREASE, LINE).
- A document containing this term is more likely to be relevant than a document that doesn't ...
- ... but words like GOOD, INCREASE and LINE are not sure indicators of relevance.
- → For frequent terms like GOOD, INCREASE, and LINE, we want positive weights . . .
- ... but lower weights than for rare terms.

Document frequency

- We want high weights for rare terms like ARACHNOCENTRIC.
- We want low (positive) weights for frequent words like GOOD, INCREASE, and LINE.
- We will use document frequency to factor this into computing the matching score.
- The document frequency is the number of documents in the collection that the term occurs in.

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idf weight

- df_t is the document frequency, the number of documents that t occurs in.
- \bullet df_t is an inverse measure of the informativeness of term t.
- We define the idf weight of term t as follows:

$$\mathsf{idf}_t = \mathsf{log}_{10} \, \frac{\mathsf{N}}{\mathsf{df}_t}$$

(N is the number of documents in the collection.)

- idf_t is a measure of the informativeness of the term.
- $[\log N/df_t]$ instead of $[N/df_t]$ to "dampen" the effect of idf
- Note that we use the log transformation for both term frequency and document frequency.

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Examples for idf

Compute idf_t using the formula: $\mathrm{idf}_t = \log_{10} \frac{1,000,000}{\mathrm{df}_t}$

term	df _t	idf _t
calpurnia	1	6
animal	100	4
sunday	1000	3
fly	10,000	2
under	100,000	1
the	1,000,000	0

Effect of idf on ranking

- idf affects the ranking of documents for queries with at least two terms.
- For example, in the query "arachnocentric line", idf weighting increases the relative weight of ARACHNOCENTRIC and decreases the relative weight of LINE.
- idf has little effect on ranking for one-term queries.

Collection frequency vs. Document frequency

word	collection frequency	document frequency
INSURANCE	10440	3997
TRY	10422	8760

- Collection frequency of t: number of tokens of t in the collection
- Document frequency of t: number of documents t occurs in
- Why these numbers?
- Which word is a better search term (and should get a higher weight)?
- This example suggests that df (and idf) is better for weighting than cf (and "icf").

tf-idf weighting

 The tf-idf weight of a term is the product of its tf weight and its idf weight.

•

$$w_{t,d} = (1 + \log \mathsf{tf}_{t,d}) \cdot \log \frac{N}{\mathsf{df}_t}$$

- tf-weight
- idf-weight
- Best known weighting scheme in information retrieval
- Note: the "-" in tf-idf is a hyphen, not a minus sign!
- Alternative names: tf.idf, tf x idf

Summary: tf-idf

- Assign a tf-idf weight for each term t in each document d: $w_{t,d} = (1 + \log \mathsf{tf}_{t,d}) \cdot \log \frac{N}{\mathsf{df}_t}$
- The tf-idf weight ...
 - ...increases with the number of occurrences within a document. (term frequency)
 - ...increases with the rarity of the term in the collection. (inverse document frequency)

Exercise: Term, collection and document frequency

Quantity	Symbol	Definition
term frequency	$tf_{t,d}$	number of occurrences of t in
		d
document frequency	df_t	number of documents in the
		collection that t occurs in
collection frequency	cf_t	total number of occurrences of
		t in the collection

- Relationship between df and cf?
- Relationship between tf and cf?
- Relationship between tf and df?

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CLEOPATRA	1	0	0	0	0	0	
MERCY	1	0	1	1	1	1	
WORSER	1	0	1	1	1	0	

Each document is represented as a binary vector $\in \{0,1\}^{|V|}$.

Count matrix

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Cleopatra	57	0	0	0	0	0	
MERCY	2	0	3	8	5	8	
WORSER	2	0	1	1	1	0	

. . .

Each document is now represented as a count vector $\in \mathbb{N}^{|V|}$.

Binary \rightarrow count \rightarrow weight matrix

	Anthony and	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth	
	Cleopatra						
Anthony	5.25	3.18	0.0	0.0	0.0	0.35	
Brutus	1.21	6.10	0.0	1.0	0.0	0.0	
Caesar	8.59	2.54	0.0	1.51	0.25	0.0	
Calpurnia	0.0	1.54	0.0	0.0	0.0	0.0	
Cleopatra	2.85	0.0	0.0	0.0	0.0	0.0	
MERCY	1.51	0.0	1.90	0.12	5.25	0.88	
WORSER	1.37	0.0	0.11	4.15	0.25	1.95	

. . .

Each document is now represented as a real-valued vector of tf-idf weights $\in \mathbb{R}^{|V|}$.

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Documents as vectors

- Each document is now represented as a real-valued vector of tf-idf weights $\in \mathbb{R}^{|V|}$.
- So we have a |V|-dimensional real-valued vector space.
- Terms are axes of the space.
- Documents are points or vectors in this space.
- Very high-dimensional: tens of millions of dimensions when you apply this to web search engines
- Each vector is very sparse most entries are zero.

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Queries as vectors

- Key idea 1: do the same for queries: represent them as vectors in the high-dimensional space
- Key idea 2: Rank documents according to their proximity to the query
- proximity = similarity
- proximity \approx negative distance
- Recall: We're doing this because we want to get away from the you're-either-in-or-out, feast-or-famine Boolean model.
- Instead: rank relevant documents higher than nonrelevant documents

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How do we formalize vector space similarity?

- First cut: (negative) distance between two points
- (= distance between the end points of the two vectors)
- Euclidean distance?
- Euclidean distance is a bad idea . . .
- ... because Euclidean distance is large for vectors of different lengths.

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Why distance is a bad idea

The Euclidean distance of \vec{q} and \vec{d}_2 is large although the distribution of terms in the query q and the distribution of terms in the document d_2 are very similar.

Questions about basic vector space setup?

Use angle instead of distance

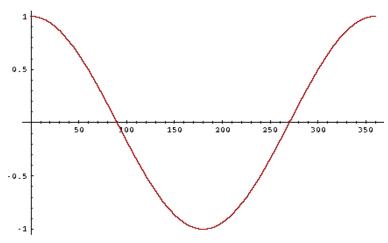
- Rank documents according to angle with query
- Thought experiment: take a document d and append it to itself. Call this document d'. d' is twice as long as d.
- "Semantically" d and d' have the same content.
- The angle between the two documents is 0, corresponding to maximal similarity . . .
- ... even though the Euclidean distance between the two documents can be quite large.

From angles to cosines

- The following two notions are equivalent.
 - Rank documents according to the angle between query and document in decreasing order
 - Rank documents according to cosine(query,document) in increasing order
- Cosine is a monotonically decreasing function of the angle for the interval $[0^{\circ}, 180^{\circ}]$

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Cosine



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Length normalization

- How do we compute the cosine?
- A vector can be (length-) normalized by dividing each of its components by its length here we use the L_2 norm: $||x||_2 = \sqrt{\sum_i x_i^2}$
- This maps vectors onto the unit sphere ...
- ... since after normalization: $||x||_2 = \sqrt{\sum_i x_i^2} = 1.0$
- As a result, longer documents and shorter documents have weights of the same order of magnitude.
- Effect on the two documents d and d' (d appended to itself) from earlier slide: they have identical vectors after length-normalization.

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Cosine similarity between query and document

$$\cos(\vec{q}, \vec{d}) = \text{SIM}(\vec{q}, \vec{d}) = \frac{\vec{q} \cdot \vec{d}}{|\vec{q}||\vec{d}|} = \frac{\sum_{i=1}^{|V|} q_i d_i}{\sqrt{\sum_{i=1}^{|V|} q_i^2} \sqrt{\sum_{i=1}^{|V|} d_i^2}}$$

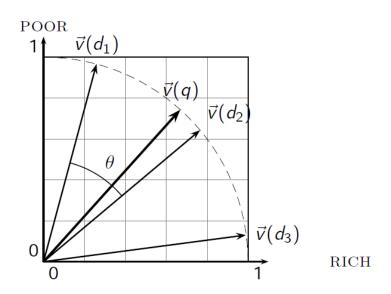
- q_i is the tf-idf weight of term i in the query.
- d_i is the tf-idf weight of term i in the document.
- $|\vec{q}|$ and $|\vec{d}|$ are the lengths of \vec{q} and \vec{d} .
- This is the cosine similarity of \vec{q} and \vec{d} or, equivalently, the cosine of the angle between \vec{q} and \vec{d} .

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Cosine for normalized vectors

- For normalized vectors, the cosine is equivalent to the dot product or scalar product.
- $\cos(\vec{q}, \vec{d}) = \vec{q} \cdot \vec{d} = \sum_i q_i \cdot d_i$
 - (if \vec{q} and \vec{d} are length-normalized).

Cosine similarity illustrated



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Cosine: Example

How similar are these novels?

SaS: Sense and Sensibility

PaP: Pride and Prejudice

WH: Wuthering

Heights

term frequencies (counts)

term	SaS	PaP	WH
AFFECTION	115	58	20
JEALOUS	10	7	11
GOSSIP	2	0	6
WUTHERING	0	0	38

Cosine: Example

term frequencies (counts)

log frequency weighting

term	SaS	PaP	WH	term	SaS	PaP	WH
AFFECTION	115	58	20	AFFECTION	3.06	2.76	2.30
JEALOUS	10	7	11	JEALOUS	2.0	1.85	2.04
GOSSIP	2	0	6	GOSSIP	1.30	0	1.78
WUTHERING	0	0	38	WUTHERING	0	0	2.58

(To simplify this example, we don't do idf weighting.)

Cosine: Example

log frequ	ency we	eighting	g	log frequency weighting				
				& cosine normalization				
term	SaS	PaP	WH	term	SaS	PaP	WH	
AFFECTION	3.06	2.76	2.30	AFFECTION	0.789	0.832	0.524	
JEALOUS	2.0	1.85	2.04	JEALOUS	0.515	0.555	0.465	
GOSSIP	1.30	0	1.78	GOSSIP	0.335	0.0	0.405	

WUTHERING

0.0

0.0

0.588

• $cos(SaS,PaP) \approx 0.789 * 0.832 + 0.515 * 0.555 + 0.335 * 0.0 + 0.0 * 0.0 \approx 0.94$.

2.58

• $cos(SaS,WH) \approx 0.79$

0

WUTHERING

0

- $cos(PaP,WH) \approx 0.69$
- Why do we have cos(SaS,PaP) > cos(SAS,WH)?

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Computing the cosine score

```
CosineScore(q)
     float Scores[N] = 0
     float Length[N]
     for each query term t
     do calculate w_{t,q} and fetch postings list for t
         for each pair(d, tf_{t,d}) in postings list
  5
         do Scores[d] += w_{t,d} \times w_{t,a}
  6
     Read the array Length
     for each d
     do Scores[d] = Scores[d]/Length[d]
10
     return Top K components of Scores[]
```

Components of tf-idf weighting

Term frequency		Docum	ent frequency	Normalization		
n (natural)	$tf_{t,d}$	n (no)	1	n (none)	1	
, - /	$1 + \log(tf_{t,d})$	t (idf)	$\log \frac{N}{df_t}$	c (cosine)	$\frac{1}{\sqrt{w_1^2 + w_2^2 + \dots + w_M^2}}$	
a (augmented)	$0.5 + rac{0.5 imes ext{t}f_{t,d}}{ ext{max}_t(ext{t}f_{t,d})}$	p (prob idf)	$\max\{0,\log rac{N-\mathrm{df}_t}{\mathrm{df}_t}\}$	u (pivoted unique)	1/u	
b (boolean)	$\begin{cases} 1 & \text{if } \operatorname{tf}_{t,d} > 0 \\ 0 & \text{otherwise} \end{cases}$			b (byte size)	$1/\mathit{CharLength}^{lpha}$, $lpha < 1$	
L (log ave)	$\frac{1 + \log(\operatorname{tf}_{t,d})}{1 + \log(\operatorname{ave}_{t \in d}(\operatorname{tf}_{t,d}))}$					

Best known combination of weighting options

Default: no weighting

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tf-idf example

- We often use different weightings for queries and documents.
- Notation: ddd.qqq
- Example: Inc.ltn
- document: logarithmic tf, no df weighting, cosine normalization
- query: logarithmic tf, idf, no normalization
- Isn't it bad to not idf-weight the document?
- Example query: "best car insurance"
- Example document: "car insurance auto insurance"

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tf-idf example: Inc.ltn

Query: "best car insurance". Document: "car insurance auto insurance".

word	query					document				product
	tf-raw	tf-wght	df	idf	weight	tf-raw	tf-wght	weight	n'lized	
auto	0	0	5000	2.3	0	1	1	1	0.52	0
best	1	1	50000	1.3	1.3	0	0	0	0	0
car	1	1	10000	2.0	2.0	1	1	1	0.52	1.04
insurance	1	1	1000	3.0	3.0	2	1.3	1.3	0.68	2.04

Key to columns: tf-raw: raw (unweighted) term frequency, tf-wght: logarithmically weighted term frequency, df: document frequency, idf: inverse document frequency, weight: the final weight of the term in the query or document, n'lized: document weights after cosine normalization, product: the product of final query weight and final document weight

$$\sqrt{1^2 + 0^2 + 1^2 + 1.3^2} \approx 1.92$$

$$1/1.92\approx 0.52\,$$

$$1.3/1.92 \approx 0.68$$

Final similarity score between query and document: $\sum_{i} w_{qi} \cdot w_{di} = 0 + 0 + 1.04 + 2.04 = 3.08$

Questions?

Summary: Ranked retrieval in the vector space model

- Represent the query as a weighted tf-idf vector
- Represent each document as a weighted tf-idf vector
- Compute the cosine similarity between the query vector and each document vector
- Rank documents with respect to the query
- ullet Return the top K (e.g., K=10) to the user

Take-away today

- Ranking search results: why it is important (as opposed to just presenting a set of unordered Boolean results)
- Term frequency: This is a key ingredient for ranking.
- Tf-idf ranking: best known traditional ranking scheme
- Vector space model: One of the most important formal models for information retrieval (along with Boolean and probabilistic models)

Resources

- Chapters 6 and 7 of IIR
- Resources at http://ifnlp.org/ir
 - Vector space for dummies
 - Exploring the similarity space (Moffat and Zobel, 2005)
 - ▶ Okapi BM25 (a state-of-the-art weighting method, 11.4.3 of IIR)

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